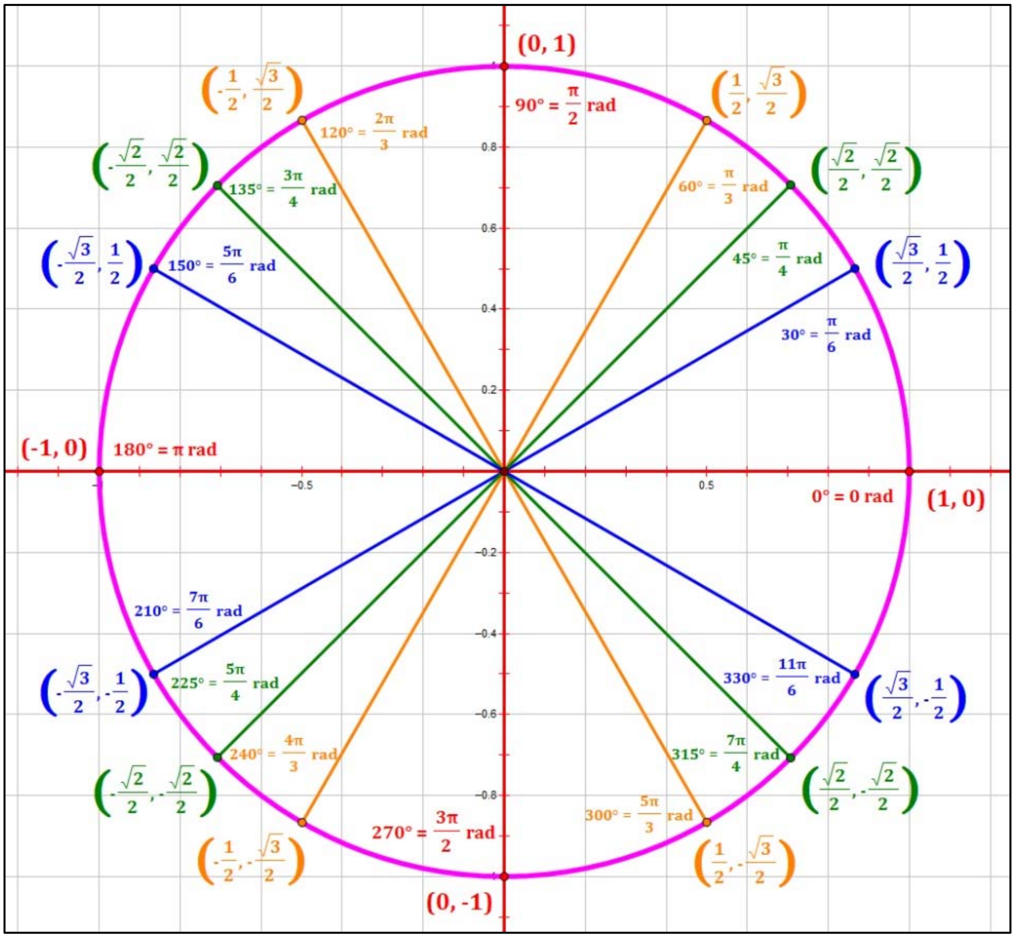
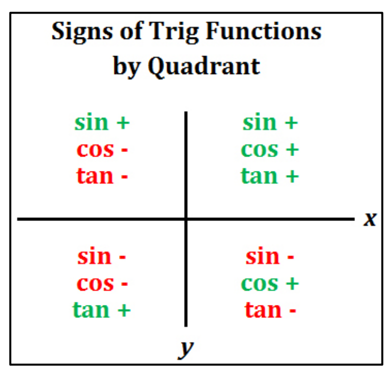
First, a couple of things to help out:



Trig Functions of Special Angles (θ)				
Radians	**Degrees**	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0^o	$\frac{\sqrt{0}}{2}=0$	$\frac{\sqrt{4}}{2}=1$	$\frac{\sqrt{0}}{\sqrt{4}}=0$
$\pi/6$	30^o	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{\sqrt{3}}=\frac{\sqrt{3}}{3}$
$\pi/4$	45^o	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{\sqrt{2}}=1$
$\pi/3$	60^o	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{3}}{\sqrt{1}}=\sqrt{3}$
$\pi/2$	90^o	$\frac{\sqrt{4}}{2}=1$	$\frac{\sqrt{0}}{2}=0$	*undefined*



SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

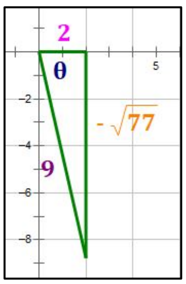
Find the exact value of the indicated trigonometric function of θ.

1) $\cos \theta = \frac{2}{9}$, $\tan \theta < 0$ Find $\sin \theta$. 1) ______________

$\cos \theta = \frac{2}{9}$ Cosine is positive in Q1 and Q4.

$\tan \theta < 0$ Tangent is negative in Q2 and Q4.

The overlap of these is Q4, so we draw our triangle in Q4, where y is negative.



$$\cos \theta = \frac{x}{r} = \frac{2}{9}$$

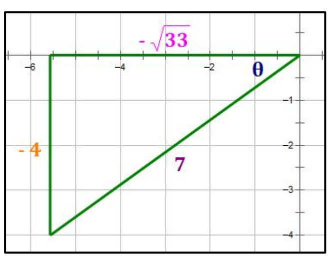
Then, the vertical leg must be:

$$y = -\sqrt{9^2 - 2^2} = -\sqrt{77}. \quad \text{Then,}$$

$$\sin \theta = \frac{y}{r} = \frac{-\sqrt{77}}{9} = \mathbf{\frac{-\sqrt{77}}{9}}$$

2) $\csc \theta = -\frac{7}{4}$, θ in quadrant III Find $\cot \theta$. 2) ______________

We are given that θ is in Q3, where x and y are negative. Recall that r is always positive!



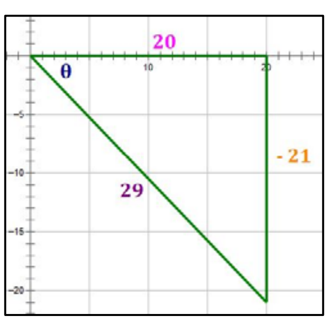
$$\csc \theta = \frac{1}{\sin \theta} = \frac{r}{y} = \frac{7}{-4}$$

Then, the horizontal leg must be:

$$x = -\sqrt{7^2 - (-4)^2} = -\sqrt{33}. \quad \text{Then,}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{x}{y} = \frac{-\sqrt{33}}{-4} = \mathbf{\frac{\sqrt{33}}{4}}$$

3) $\cos \theta = \frac{20}{29}$, $\frac{3\pi}{2} < \theta < 2\pi$ Find $\cot \theta$. 3) ______________



$\frac{3\pi}{2} < \theta < 2\pi$. Therefore θ is in $Q4$, where y is negative.

$$\cos \theta = \frac{x}{r} = \frac{20}{29}$$

Then, the vertical leg must be:

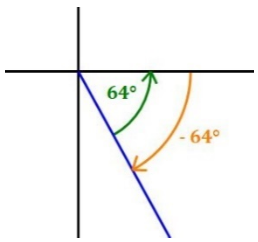
$$y = -\sqrt{29^2 - (20)^2} = -21. \quad \text{Then,}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{x}{y} = \frac{20}{-21} = -\mathbf{\frac{20}{21}}$$

Reference Angle Formulas by Quadrant (reference angles are always less than 90° or $\frac{\pi}{2}$ radians)		
Quadrant	**In Degrees**	**In Radians**
Q1	$\rho = \theta$	$\rho = \theta$
Q2	$\rho = 180° - \theta$	$\rho = \pi - \theta$
Q3	$\rho = \theta - 180°$	$\rho = \theta - \pi$
Q4	$\rho = 360° - \theta$	$\rho = 2\pi - \theta$
Notation: ρ is the reference angle of angle θ. To use these formulas, θ must be positive and less than 360° or 2π radians.		

Find the reference angle for the given angle.

4) $-64°$ 4) ________________



The reference angle is the angle between the original angle's terminal side and the x-axis. It must be between 0 and $\frac{\pi}{2}$ radians ($0°$ and $90°$).

For negative angles in Q4, this means that the reference angle is the absolute value of the negative angle: $\rho = |-64°| = \mathbf{64°}$

Alternative: Add 360° to get a positive angle less than 360°, then use the rule above for Q4 to obtain the reference angle:

$-64° + 360° = 296°$, an angle in Q4. Then,

$\rho = 360° - 296° = \mathbf{64°}$

5) $-\frac{2\pi}{3}$ 5) ________________

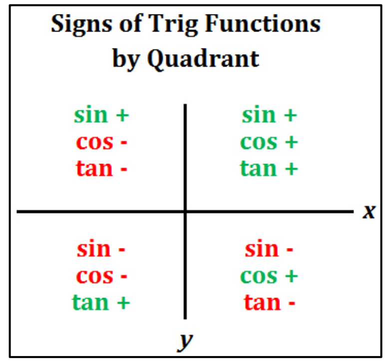
First, find the positive coterminal angle less than 2π radians.

$-\frac{2\pi}{3} + 2\pi = \frac{4}{3}\pi$. This angle is in Quadrant 3. Then, the reference angle is:

$$\rho = \frac{4}{3}\pi - \pi = \boldsymbol{\frac{\pi}{3}}$$

Use reference angles to find the exact value of the expression. Do not use a calculator.

6) $\csc \frac{4\pi}{3}$ 6) ______________



The angle given, $\frac{4\pi}{3}$, is positive and less than 2π radians, so we can work with it directly. It is in Quadrant 3. Then, the reference angle is:

$$\rho = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$$

Next, using the reference angle:

$$\csc\left(\frac{\pi}{3}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Finally, consider the sign pattern of the cosecant function. It follows the sign pattern of the sine function, which is negative in Quadrant 3. Therefore,

$$\csc\left(\frac{4\pi}{3}\right) = -\frac{\mathbf{2\sqrt{3}}}{\mathbf{3}}$$

7) $\tan \frac{7\pi}{6}$ 7) ______________

This angle is in Quadrant 3. Then, the reference angle is:

$$\rho = \frac{7}{6}\pi - \pi = \frac{\pi}{6}$$

Next, using the reference angle:

$$\tan\left(\frac{\pi}{6}\right) = \frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Finally, consider the sign pattern of the Tangent function, which is positive in Quadrant 3. Therefore,

$$\tan\left(\frac{7\pi}{6}\right) = \frac{\mathbf{\sqrt{3}}}{\mathbf{3}}$$

8) $\sin \frac{4\pi}{3}$ 8) ______________

The angle given, $\frac{4\pi}{3}$, is in Quadrant 3. Then, the reference angle is:

$$\rho = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$$

Next, using the reference angle:

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Finally, consider the sign pattern of the sine function, which is negative in Quadrant 3. Therefore,

$$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

9) $\sec \frac{-5\pi}{4}$ 9) ______________

First, find the positive coterminal angle less than 2π radians.

$-\frac{5\pi}{4} + 2\pi = \frac{3\pi}{4}$. This angle is in Quadrant 2. So, the reference angle is:

$$\rho = \pi - \frac{3}{4}\pi = \frac{\pi}{4}$$

Next, using the reference angle:

$$\sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Finally, consider the sign pattern of the secant function, which is the same as that of the cosine function. $-\frac{5\pi}{4}$ is in Quadrant 2, where the cosine function is negative. Therefore,

$$\sec\left(-\frac{5\pi}{4}\right) = -\sqrt{2}$$

Determine the amplitude or period as requested.

10) Amplitude of $y = 2 \sin 5x$ 10) ______________

Amplitude is the measure of the distance of peaks and troughs from the midline (i.e., center) of a *sine or cosine function*; amplitude is always positive. For the general function, $f(x) = \mathrm{A} \cdot \sin(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, amplitude $= |\mathrm{A}|$.

In the case of the function $y = 2 \sin 5x$, the amplitude is $|2|$. Therefore, the amplitude of this function is **2**.

11) Period of $y = -4 \sin 6\pi x$ 11) ______________

Period is the horizontal width of a single cycle or wave, i.e., the distance it travels before it repeats. Every trigonometric function has a period. The periods of the *parent functions* are as follows: for sine, cosine, secant and cosecant, period $= 2\pi$; for tangent and cotangent, period $= \pi$.

For the general function, $f(x) = \mathrm{A} \cdot \mathrm{trig}(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$,

$$\text{period} = \frac{\text{parent function period}}{\mathrm{B}}$$

In the case of the function $y = -4\sin(6\pi x)$, the parent function period is 2π and $B = 6\pi$.

Therefore, the period of this function is: $\frac{\boldsymbol{2\pi}}{6\pi} = \frac{\mathbf{1}}{\mathbf{3}}$ **radians.**

Determine the phase shift of the function.

12) $y = 5 \sin\left(4x - \frac{\pi}{2}\right)$ 12) ______________

Phase shift is how far the function has been shifted horizontally (left or right) from its parent function. For the general function, $f(x) = \mathrm{A} \cdot \mathrm{trig}(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$,

$$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}}$$

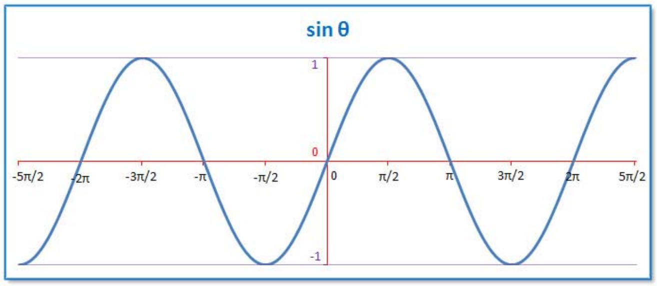
A positive phase shift indicates a shift to the right relative to the graph of the parent function; a negative phase shift indicates a shift to the left relative to the graph of the parent function.

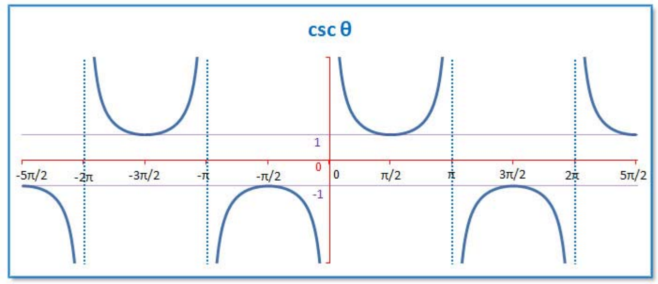
A trick for calculating the phase shift is to set the argument of the trigonometric function equal to zero: $(\mathrm{B}x - \mathrm{C}) = 0$, and solve for x. The resulting value of x is the phase shift of the function.

In the case of the function $y = 5\sin\left(4x - \frac{\pi}{2}\right)$, set $\left(4x - \frac{\pi}{2}\right) = 0$ and solve for x:

$$\left(4x - \frac{\pi}{2}\right) = 0 \qquad \Rightarrow \qquad 4x = \frac{\pi}{2} \qquad \Rightarrow \qquad \boldsymbol{x = \frac{\pi}{8}} \ \textbf{radians.}$$

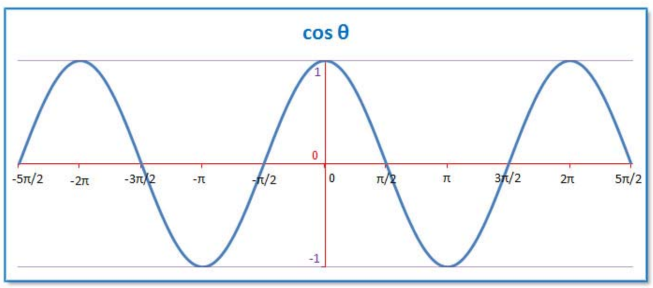
Graphs of Basic (Parent) Trigonometric Functions

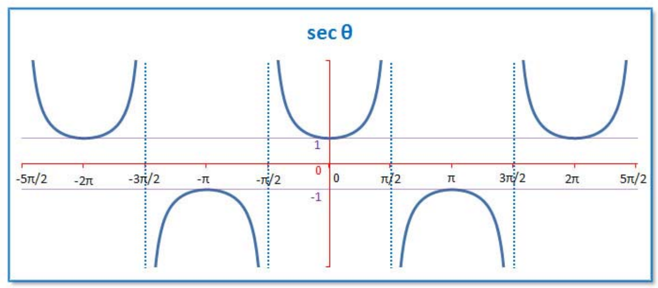




The sine and cosecant functions are reciprocals. So:

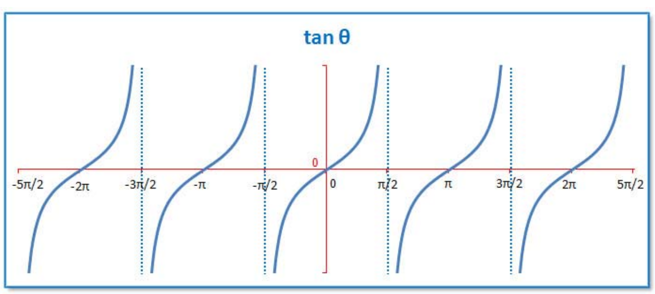
$$\sin\theta = \frac{1}{\csc\theta} \quad \text{and} \quad \csc\theta = \frac{1}{\sin\theta}$$

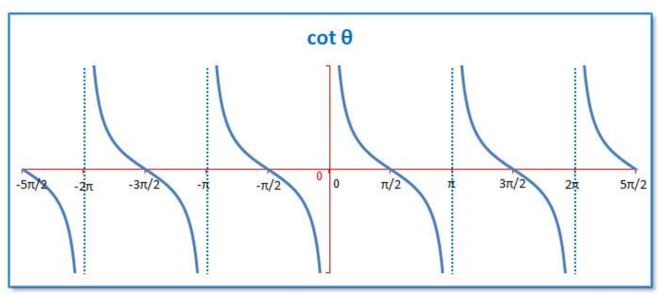




The cosine and secant functions are reciprocals. So:

$$\cos\theta = \frac{1}{\sec\theta} \quad \text{and} \quad \sec\theta = \frac{1}{\cos\theta}$$





The tangent and cotangent functions are reciprocals. So:

$$\tan\theta = \frac{1}{\cot\theta} \quad \text{and} \quad \cot\theta = \frac{1}{\tan\theta}$$

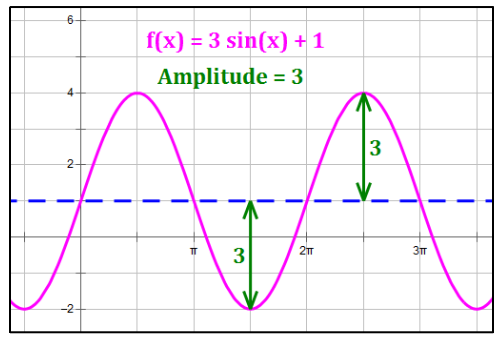
Characteristics of Trigonometric Function Graphs

All trigonometric functions are periodic, meaning that they repeat the pattern of the curve (called a cycle) on a regular basis. The key characteristics of each curve, along with knowledge of the parent curves are sufficient to graph many trigonometric functions. Let's consider the general function:

$$f(x) = \mathrm{A} \cdot trig(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$$

where A, B, C and D are constants and "$trig$" is any of the six trigonometric functions (sine, cosine, tangent, cotangent, secant, cosecant).

Amplitude



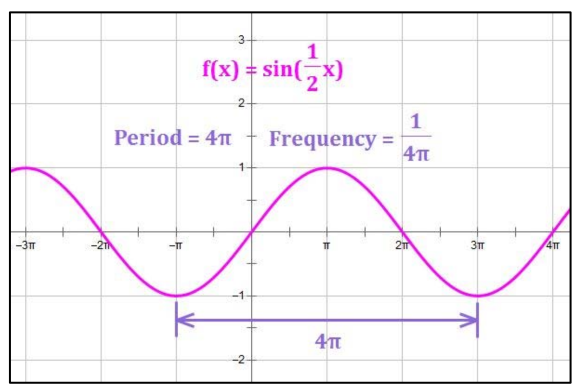
Amplitude is the measure of the distance of peaks and troughs from the midline (i.e., center) of a *sine or cosine function*; amplitude is always positive. The other four functions do not have peaks and troughs, so they do not have amplitudes. For the general function, $f(x)$, defined above, $\text{amplitude} = |\mathrm{A}|$.

Period

Period is the horizontal width of a single cycle or wave, i.e., the distance it travels before it repeats. Every trigonometric function has a period. The periods of the *parent functions* are as follows: for sine, cosine, secant and cosecant, $\text{period} = 2\pi$; for tangent and cotangent, $\text{period} = \pi$.

For the general function, $f(x)$, defined above,

$$\text{period} = \frac{\text{parent function period}}{\mathrm{B}}.$$



Frequency

Frequency is most useful when used with the sine and cosine functions. It is the reciprocal of the period, i.e.,

$$\text{frequency} = \frac{1}{\text{period}}.$$

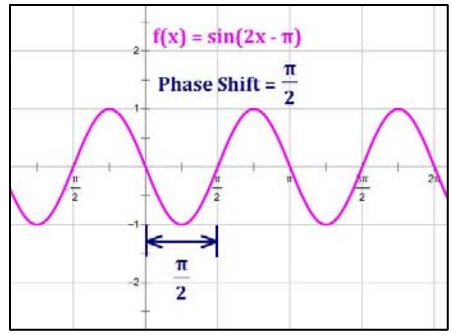
Frequency is typically discussed in relation to the sine and cosine functions when considering harmonic motion or waves. In Physics, frequency is typically measured in Hertz, i.e., cycles per second. 1 Hz $=$ 1 cycle per second.

For the general sine or cosine function, $f(x)$, defined above, $\text{frequency} = \dfrac{\mathrm{B}}{2\pi}$.

Phase Shift

Phase shift is how far has the function been shifted horizontally (left or right) from its parent function. For the general function, $f(x)$, defined above,

$$\text{phase shift } = \frac{C}{B}.$$

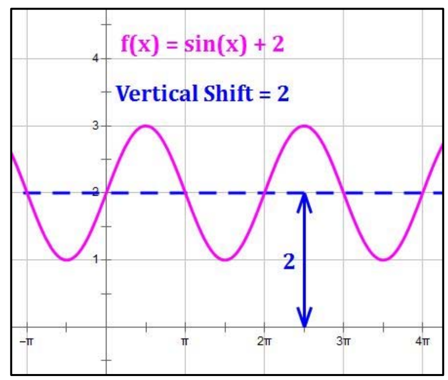


A positive phase shift indicates a shift to the right relative to the graph of the parent function; a negative phase shift indicates a shift to the left relative to the graph of the parent function.

A trick for calculating the phase shift is to set the argument of the trigonometric function equal to zero: $(Bx - C) = 0$, and solve for x. The resulting value of x is the phase shift of the function.

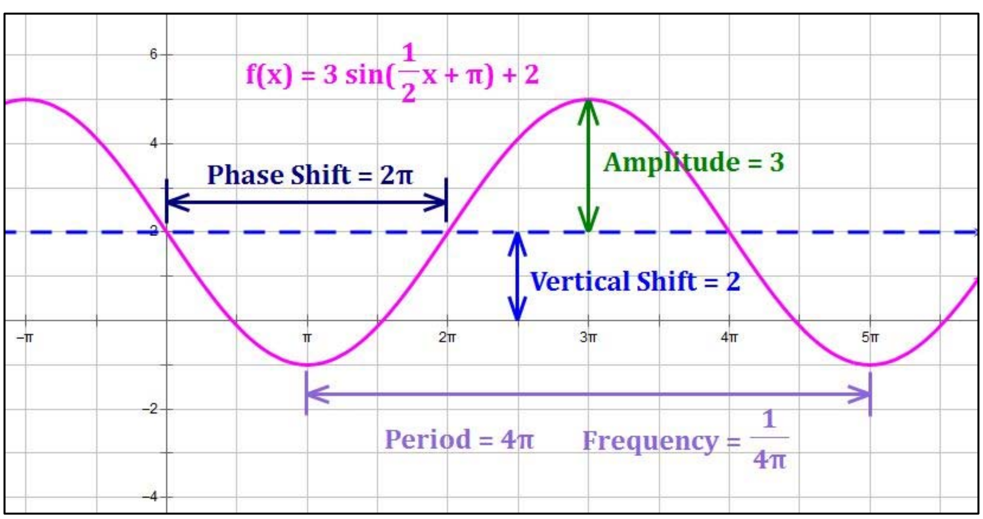
Vertical Shift

Vertical shift is the vertical distance that the midline of a curve lies above or below the midline of its parent function (i.e., the x-axis). For the general function, $f(x)$, defined above, $\text{vertical shift} = D.$ The value of D may be positive, indicating a shift upward, or negative, indicating a shift downward relative to the graph of the parent function.



Putting it All Together

The illustration below shows how all of the items described above combine in a single graph.



Graph the function, Identify the amplitude, period and phase shift (if any) .

13) y = -3 sin 3x 13) ______________

Note: this approach uses transformations. An alternative approach for each graph that uses the plotted-point approach is shown on the page following the transformation approach.

This section requests that you identify the amplitude, period, and phase shift of each function that you want to graph. You should do this anyway with every graph of a Trig function.

$y = -3\sin 3x$

Relative to the general function, $f(x) = \mathrm{A}\cdot\sin(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = -3, B = 3, C = 0, D = 0.$ Then,

$\text{amplitude} = |\mathrm{A}| = |-3| = 3$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{2\pi}{3}$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = 0$

"–" in front of the function indicates a reflection over the x-axis.

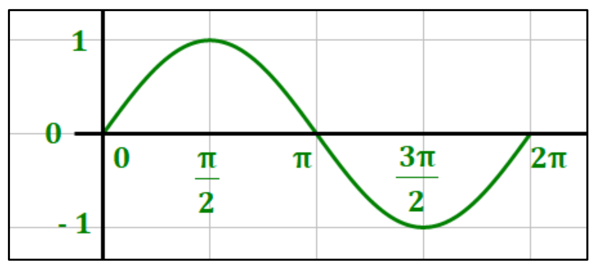
The periods of the *parent functions* are as follows:

- sin, cos, sec, csc: 2π
- tan, cot: π

Start: Graph the parent function

$y = \sin x$

Changes in successive graphs are shown in magenta in the following steps.

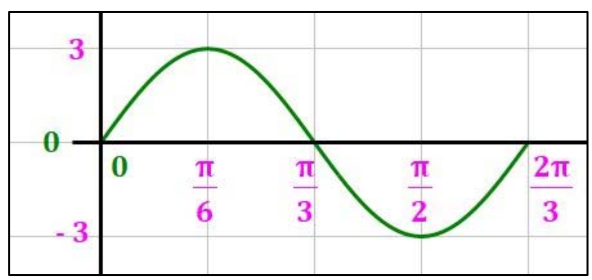


Adjust the amplitude:

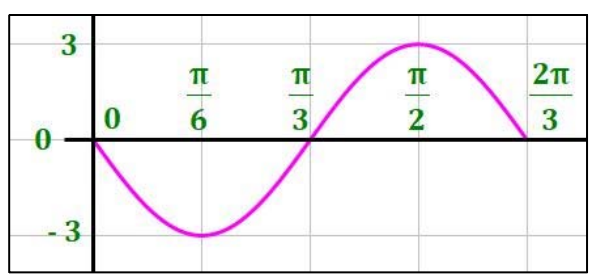
- Change amplitude from 1 to $|A| = 3$
- Change y-axis labels

Adjust the period:

- Parent period is $[0, 2\pi]$
- Divide x-axis labels by $B = 3$



Reflect the curve over the x-axis because of the minus sign in the front of the equation.



Alternative approach:

Graph the function, Identify the amplitude, period and phase shift (if any) .

13) $y = -3 \sin 3x$ 13) ______________

This section requests that you identify the amplitude, period, and phase shift of each function that you want to graph. You should do this anyway with every graph of a Trig function.

$y = -3 \sin 3x$

Relative to the general function, $f(x) = \mathrm{A} \cdot \sin(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = -3, B = 3, C = 0, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |-3| = 3$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{2\pi}{3}$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = 0$

"$-$" in front of the function indicates a reflection over the x-axis.

The periods of the *parent functions* are as follows:

- sin, cos, sec, csc: 2π
- tan, cot: π

Let's get five points. Points of interest on the parent function exist at: $x = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$.

Points for the plot: Since $B = 3$, divide each x of interest in the parent function by 3 and add the phase shift (0 in this problem). The resulting x-values of interest are: $x = \left\{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}\right\}$. Note that the last x-value should be equal to the period plus the phase shift.

Now, let's find y-values for each x-value:

$x = 0 \quad y = -3\sin(3 \cdot 0) = -3 \sin 0 = -3 \cdot 0 = 0 \quad$ Point: $(0, 0)$

$x = \frac{\pi}{6} \quad y = -3\sin\left(3 \cdot \frac{\pi}{6}\right) = -3 \sin \frac{\pi}{2} = -3 \cdot 1 = -3 \quad$ Point: $\left(\frac{\pi}{6}, -3\right)$

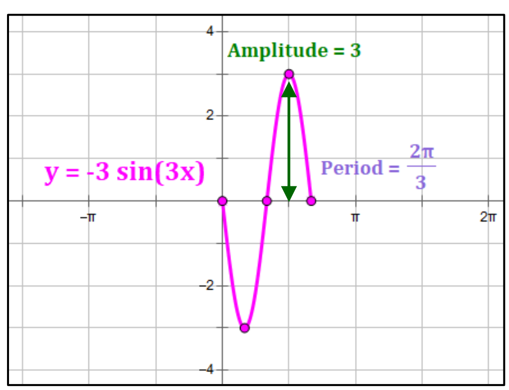
$x = \frac{\pi}{3} \quad y = -3\sin\left(3 \cdot \frac{\pi}{3}\right) = -3 \sin \pi = -3 \cdot 0 = 0 \quad$ Point: $\left(\frac{\pi}{3}, 0\right)$

$x = \frac{\pi}{2} \quad y = -3\sin\left(3 \cdot \frac{\pi}{2}\right) = -3 \sin \frac{3\pi}{2} = -3 \cdot (-1) = 3 \quad$ Point: $\left(\frac{\pi}{2}, 3\right)$

$x = \frac{2\pi}{3} \quad y = -3\sin\left(3 \cdot \frac{2\pi}{3}\right) = -3 \sin 2\pi = -3 \cdot 0 = 0 \quad$ Point: $\left(\frac{2\pi}{3}, 0\right)$

Plot the points and run a curve through them.

This will show a single wave of the function.



14) y = 3 sin (2x + π) 14) ______________

$y = 3\sin(2x + \pi)$

Relative to the general function, $f(x) = \mathrm{A} \cdot \sin(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 3, B = 2, C = -\pi, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |3| = 3$

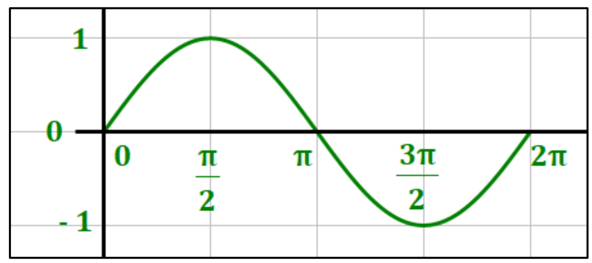
$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{2\pi}{2} = \pi$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = -\dfrac{\pi}{2} \Rightarrow \dfrac{\pi}{2}$ to the left

Start: Graph the parent function

$y = \sin x$

Changes in successive graphs are shown in magenta in the following steps.

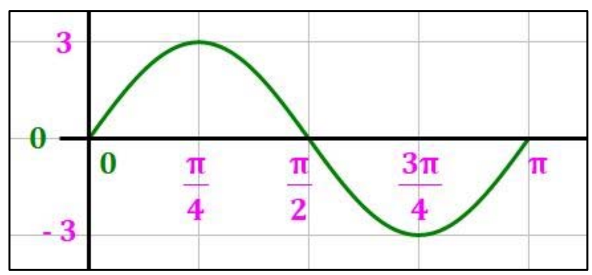


Adjust the amplitude:

- Change amplitude from 1 to $|A| = 3$
- Change y-axis labels

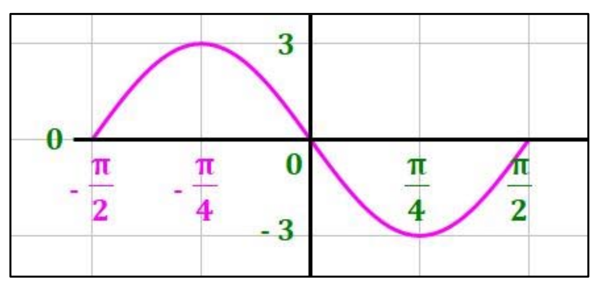
Adjust the period:

- Parent period is $[0, 2\pi]$
- Divide x-axis labels by $B = 2$



Phase shift the function $\dfrac{\pi}{2}$ to the left.

Also, adjust the x-axis labels to reflect the shift (subtract $\dfrac{\pi}{2}$ from each x-axis label and position the labels correctly on the graph).



Alternative approach:

14) $y = 3\sin(2x + \pi)$ 14) ______________

$y = 3\sin(2x + \pi)$

Relative to the general function, $f(x) = \mathrm{A} \cdot \sin(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 3, B = 2, C = -\pi, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |3| = 3$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{2\pi}{2} = \pi$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = -\dfrac{\pi}{2} \Rightarrow \dfrac{\pi}{2}$ to the left

Let's get five points. Points of interest on the parent function exist at: $x = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$.

Points for the plot: Since $B = 2$, divide each x of interest in the parent function by 2 and add the phase shift ($-\frac{\pi}{2}$ in this problem). The resulting x-values of interest are: $x = \left\{-\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}\right\}$. Note that the last x-value should be equal to the period plus the phase shift.

Now, let's find y-values for each x-value:

$x = -\frac{\pi}{2}$ $\quad y = 3\sin\left[\left(2 \cdot -\frac{\pi}{2}\right) + \pi\right] = 3\sin 0 = 3 \cdot 0 = 0$ $\quad$ Point: $(-\frac{\pi}{2}, 0)$

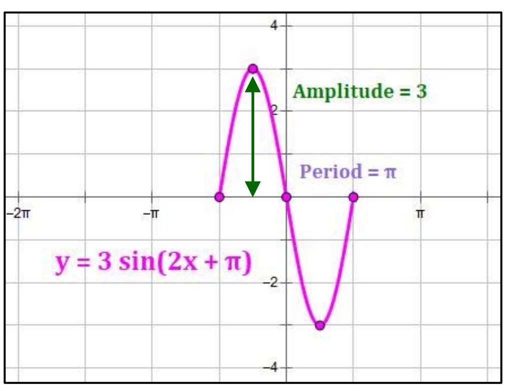
$x = -\frac{\pi}{4}$ $\quad y = 3\sin\left[\left(2 \cdot -\frac{\pi}{4}\right) + \pi\right] = 3\sin\frac{\pi}{2} = 3 \cdot 1 = 3$ $\quad$ Point: $\left(-\frac{\pi}{4}, 3\right)$

$x = 0$ $\quad y = 3\sin[(2 \cdot 0) + \pi] = 3\sin\pi = 3 \cdot 0 = 0$ $\quad$ Point: $(0, 0)$

$x = \frac{\pi}{4}$ $\quad y = 3\sin\left[\left(2 \cdot \frac{\pi}{4}\right) + \pi\right] = 3\sin\frac{3\pi}{2} = 3 \cdot (-1) = -3$ $\quad$ Point: $\left(\frac{\pi}{4}, -3\right)$

$x = \frac{\pi}{2}$ $\quad y = 3\sin\left[\left(2 \cdot \frac{\pi}{2}\right) + \pi\right] = 3\sin 2\pi = 3 \cdot 0 = 0$ $\quad$ Point: $\left(\frac{\pi}{2}, 0\right)$

Plot the points and run a curve through them. This will show a single wave of the function.



15) y = 2 cos 3x 15) _______________

$y = 2\cos 3x$

Relative to the general function, $f(x) = \mathrm{A} \cdot \cos(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 2, B = 3, C = 0, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |2| = 2$

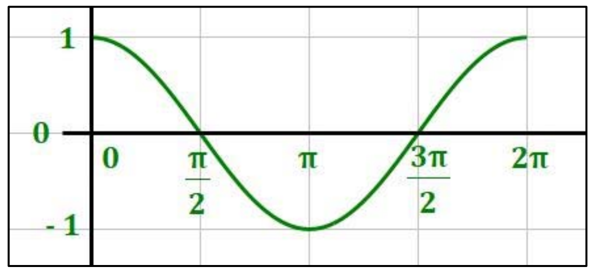
$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{2\pi}{3}$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = 0$

Start: Graph the parent function

$y = \cos x$

Changes in successive graphs are shown in magenta in the following steps.

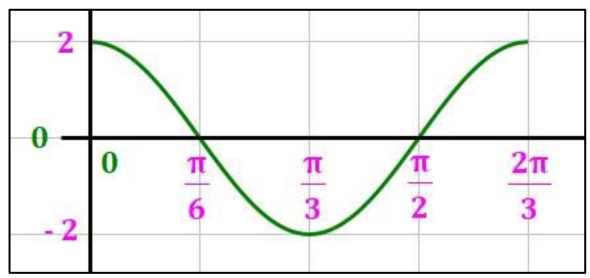


Adjust the amplitude:

- Change amplitude from 1 to $|A| = 2$
- Change y-axis labels

Adjust the period:

- Parent period is $[0, 2\pi]$
- Divide x-axis labels by $B = 3$



Alternative approach:

15) y = 2 cos 3x

15) ______________

$y = 2\cos 3x$

Relative to the general function, $f(x) = \mathrm{A} \cdot \cos(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 2, B = 3, C = 0, D = 0$. Then,

$$\text{amplitude} = |\mathrm{A}| = |2| = 2$$

$$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{2\pi}{3}$$

$$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = 0$$

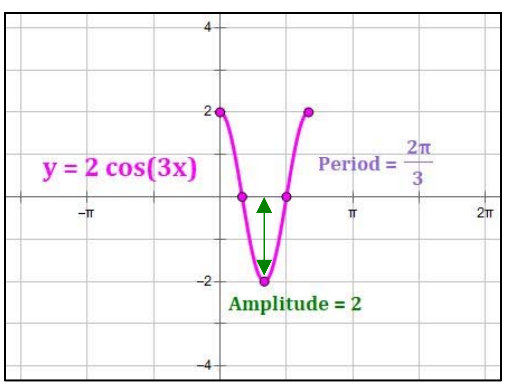
Let's get five points. Points of interest on the parent function exist at: $x = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$.

Points for the plot: Since $B = 3$, divide each x of interest in the parent function by 3 and add the phase shift (0 in this problem). The resulting x-values of interest are: $x = \left\{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}\right\}$. Note that the last x-value should be equal to the period plus the phase shift.

Now, let's find y-values for each x-value:

$x = 0$	$y = 2\cos(3 \cdot 0) = 2\cos 0 = 2 \cdot 1 = 2$	Point: $(0, 2)$
$x = \frac{\pi}{6}$	$y = 2\cos\left(3 \cdot \frac{\pi}{6}\right) = 2\cos\frac{\pi}{2} = 2 \cdot 0 = 0$	Point: $\left(\frac{\pi}{6}, 0\right)$
$x = \frac{\pi}{3}$	$y = 2\cos\left(3 \cdot \frac{\pi}{3}\right) = 2\cos\pi = 2 \cdot (-1) = -2$	Point: $\left(\frac{\pi}{3}, -2\right)$
$x = \frac{\pi}{2}$	$y = 2\cos\left(3 \cdot \frac{\pi}{2}\right) = 2\cos\frac{3\pi}{2} = 2 \cdot 0 = 0$	Point: $\left(\frac{\pi}{2}, 0\right)$
$x = \frac{2\pi}{3}$	$y = 2\cos\left(3 \cdot \frac{2\pi}{3}\right) = 2\cos 2\pi = 2 \cdot 1 = 2$	Point: $\left(\frac{2\pi}{3}, 2\right)$

Plot the points and run a curve through them. This will show a single wave of the function.



16) $y = 4\cos\left(x - \frac{\pi}{4}\right)$ 16) ______________

$y = 4\cos\left(x - \frac{\pi}{4}\right)$

Relative to the general function, $f(x) = \mathrm{A}\cdot\cos(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 4, B = 1, C = \frac{\pi}{4}, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |4| = 4$

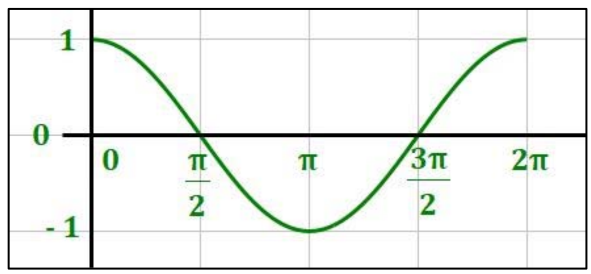
$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{2\pi}{1} = 2\pi$

$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = \frac{\pi}{4} \Rightarrow \frac{\pi}{4}$ to the right

Start: Graph the parent function

$y = \cos x$

Changes in successive graphs are shown in magenta in the following steps.

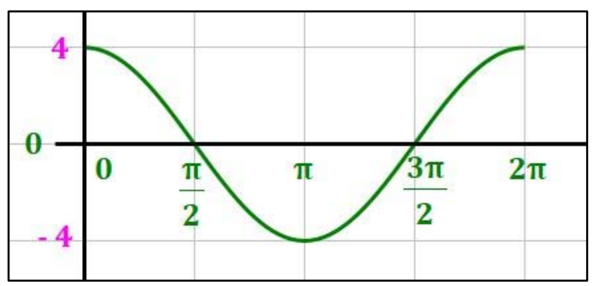


Adjust the amplitude:

- Change amplitude from 1 to $|A| = 4$
- Change y-axis labels

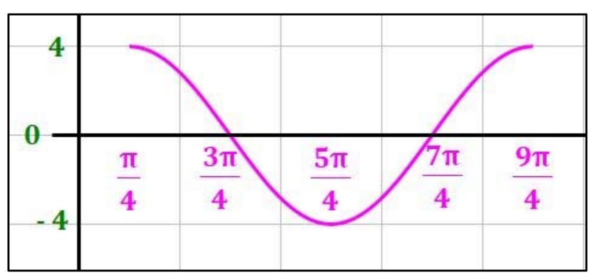
There is no change to the length of the period because $B = 1$

- Parent period is $[0, 2\pi]$



Phase shift the function $\frac{\pi}{4}$ to the right

Also, adjust the x-axis labels to reflect the shift (add $\frac{\pi}{4}$ to each x-axis label and position the labels correctly on the graph).



Alternative approach:

16) $y = 4\cos\left(x - \frac{\pi}{4}\right)$ 16) ____________

$y = 4\cos\left(x - \frac{\pi}{4}\right)$

Relative to the general function, $f(x) = \mathrm{A} \cdot \cos(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 4, B = 1, C = \frac{\pi}{4}, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |4| = 4$

$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{2\pi}{1} = 2\pi$

$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = \frac{\pi}{4} \Rightarrow \frac{\pi}{4}$ to the right

Let's get five points. Points of interest on the parent function exist at: $x = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$.

Points for the plot: Since $B = 1$, start with each x of interest in the parent function add the phase shift ($\frac{\pi}{4}$ in this problem). The resulting x-values of interest are: $x = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}\right\}$. Note that the last x-value should be equal to the period plus the phase shift.

Now, let's find y-values for each x-value:

$x = \frac{\pi}{4}$ $\quad y = 4\cos\left[\left(\frac{\pi}{4}\right) - \frac{\pi}{4}\right] = 4\cos 0 = 4 \cdot 1 = 4$ $\quad$ Point: $\left(\frac{\pi}{4}, 4\right)$

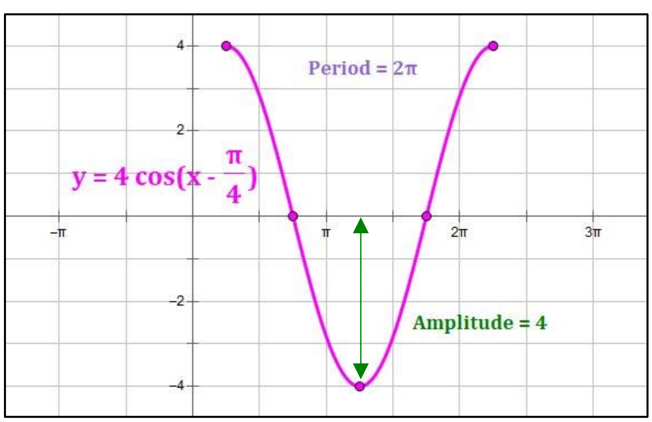
$x = \frac{3\pi}{4}$ $\quad y = 4\cos\left[\left(\frac{3\pi}{4}\right) - \frac{\pi}{4}\right] = 4\cos\frac{\pi}{2} = 4 \cdot 0 = 0$ $\quad$ Point: $\left(\frac{3\pi}{4}, 0\right)$

$x = \frac{5\pi}{4}$ $\quad y = 4\cos\left[\left(\frac{5\pi}{4}\right) - \frac{\pi}{4}\right] = 4\cos\pi = 4 \cdot (-1) = -4$ $\quad$ Point: $\left(\frac{5\pi}{4}, -4\right)$

$x = \frac{7\pi}{4}$ $\quad y = 4\cos\left[\left(\frac{7\pi}{4}\right) - \frac{\pi}{4}\right] = 4\cos\frac{3\pi}{2} = 4 \cdot 0 = 0$ $\quad$ Point: $\left(\frac{7\pi}{4}, 0\right)$

$x = \frac{9\pi}{4}$ $\quad y = 4\cos\left[\left(\frac{9\pi}{4}\right) - \frac{\pi}{4}\right] = 4\cos 2\pi = 4 \cdot 1 = 4$ $\quad$ Point: $\left(\frac{9\pi}{4}, 4\right)$

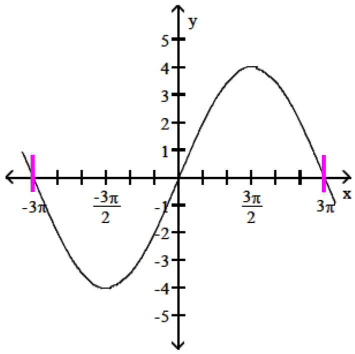
Plot the points and run a curve through them. This will show a single wave of the function.



MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find an equation for the graph.

17) 17) _______



This looks like a sine function because it crosses the y-axis at $(0, 0)$.

The function appears to have no phase shift because it is symmetric about the origin (i.e., it is an odd function). Therefore, $C = 0$.

A) $y = 3 \sin 4x$ B) $y = 4 \sin 3x$ C) $y = 3 \sin \frac{1}{4}x$ D) $y = 4 \sin \frac{1}{3}x$

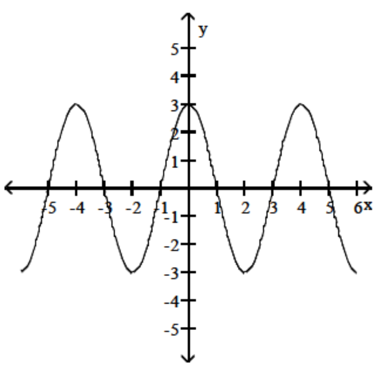
The period of the function (the distance between the vertical magenta marks on the diagram) is 6π.

Then, $\frac{\text{parent function period}}{\text{B}} = \frac{2\pi}{B} = 6\pi \quad \Rightarrow \quad B = \frac{2\pi}{6\pi} = \frac{1}{3}$.

The function has an amplitude of 4 because it oscillates between -4 and 4. Furthermore, there is no reflection over the x-axis, so A is positive. Therefore $A = 4$.

Put it all together to get: $\boldsymbol{y = 4 \sin\left(\frac{1}{3}x\right)}$ **Answer D**

18) 18) _______



This looks like a cosine function because it crosses the y-axis at its maximum value.

The function appears to have no phase shift because it is symmetric about y-axis (i.e., it is an even function). Therefore, $C = 0$.

A) $y = 3 \cos \frac{\pi}{2}x$ B) $y = 3 \cos 2\pi x$ C) $y = 2 \cos \frac{\pi}{3}x$ D) $y = 2 \cos 3\pi x$

The period of the function (the distance between the peaks on the diagram is 4. Then,

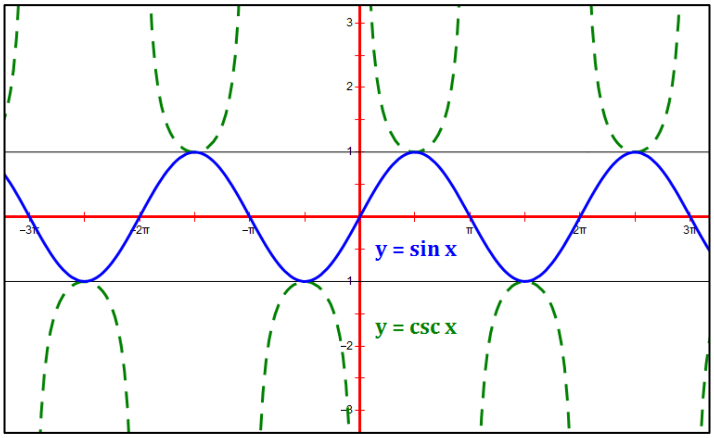
$\frac{\text{parent function period}}{\text{B}} = \frac{2\pi}{B} = 4 \quad \Rightarrow \quad B = \frac{2\pi}{4} = \frac{\pi}{2}$.

The function has an amplitude of 3 because it oscillates between -3 and 3. Furthermore, there is no reflection over the x-axis, so A is positive. Therefore $A = 3$.

Put it all together to get: $\boldsymbol{y = 3 \cos\left(\frac{\pi}{2}x\right)}$ **Answer A**

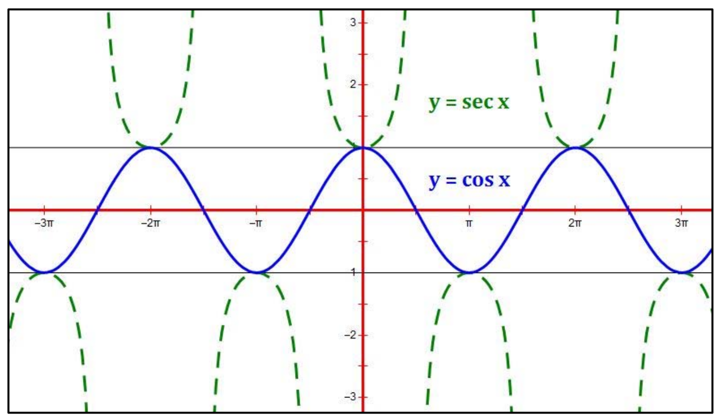
Graphs of Related Parent Trigonometric Functions

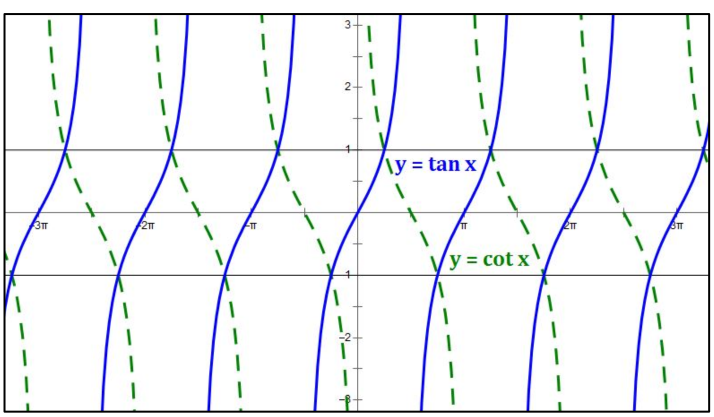
It is instructive to view the parent trigonometric functions on the same axes as their reciprocals. Identifying patterns between the two functions can be helpful in graphing them.



Looking at the **sine** and **cosecant** functions, we see that they intersect at their maximum and minimum values (i.e., when $y = 1$). The vertical asymptotes (not shown) of the cosecant function occur when the sine function is zero.

Looking at the **cosine** and **secant** functions, we see that they intersect at their maximum and minimum values (i.e., when $y = 1$). The vertical asymptotes (not shown) of the secant function occur when the cosine function is zero.





Looking at the **tangent** and **cotangent** functions, we see that they intersect when $\sin x = \cos x$ (i.e., at $x = \frac{\pi}{4} + n\pi$, n an integer). The vertical asymptotes (not shown) of the each function occur when the other function is zero.

Graph the function.

19) y = 2 tan 4x   19) ______________

$y = 2\tan 4x$

Relative to the general function, $f(x) = \mathrm{A}\cdot\tan(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 2, B = 4, C = 0, D = 0.$ Then,

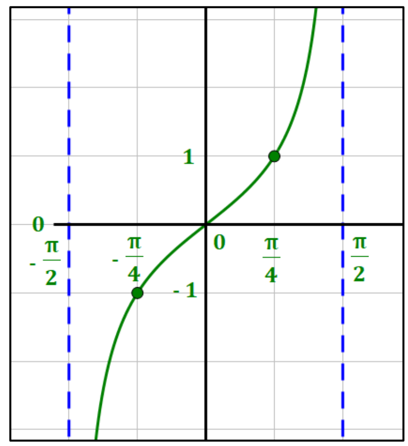
$\text{amplitude} = |\mathrm{A}| = |2| = 2$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{\pi}{4}$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = 0$

The periods of the *parent functions* are as follows:

- sin, cos, sec, csc: 2π
- tan, cot: π

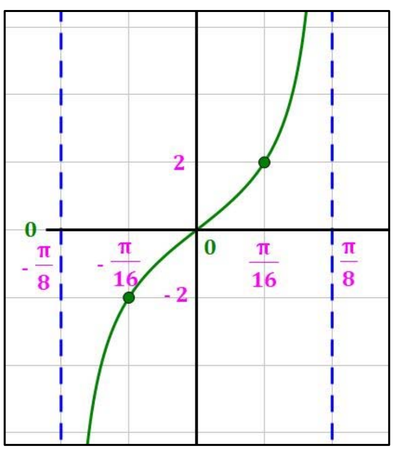


Start: Graph the parent function and its asymptotes.

$y = \tan x$

Also, plot the points $\left(-\frac{\pi}{4}, -1\right)$, $\left(\frac{\pi}{4}, 1\right)$

Changes in successive graphs are shown in magenta in the following steps.



Adjust the amplitude:

- Change the amplitude from 1 to $|A| = 2$ by multiplying the y-axis labels by 2.

Adjust the period:

- Parent period is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- Divide x-axis labels by $B = 4$

Alternative approach:

Graph the function.

19) $y = 2\tan 4x$ 19) ______________

$y = 2\tan 4x$

Relative to the general function, $f(x) = \mathrm{A}\cdot\tan(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 2, B = 4, C = 0, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |2| = 2$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{\pi}{4}$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = 0$

The periods of the *parent functions* are as follows:

- sin, cos, sec, csc: 2π
- tan, cot: π

Let's get two asymptotes and three points. Asymptotes of the parent function are: $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$.

Points of interest on the parent function exist at: $x = \left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$.

Asymptotes for the plot: Since $B = 4$, divide each asymptote value of the parent function by 4 and add the phase shift (0 in this problem). The resulting asymptotes are:

$x = -\dfrac{\pi}{8}$ and $x = \dfrac{\pi}{8}$

Note that the distance between the asymptotes should be equal to the period.

Points for the plot: Since $B = 4$, divide each x of interest in the parent function by 4 and add the phase shift (0 in this problem). The resulting x-values of points of interest are: $x = \left\{-\frac{\pi}{16}, 0, \frac{\pi}{16}\right\}$.

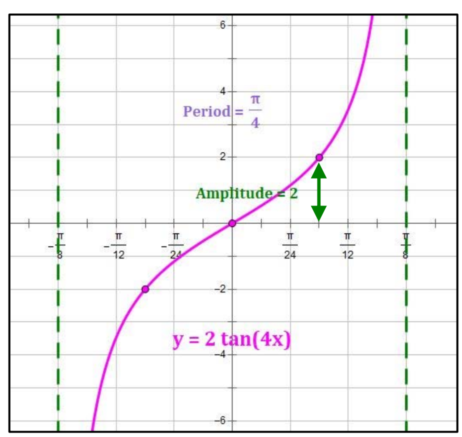
Now, let's find y-values for each x-value:

$x = -\frac{\pi}{16}$ $\quad y = 2\tan\left(4\cdot\left(-\frac{\pi}{16}\right)\right) = 2\tan\left(-\frac{\pi}{4}\right) = 2\cdot(-1) = -2$ $\quad$ Point: $\left(-\frac{\pi}{16}, -2\right)$

$x = 0$ $\quad y = 2\tan(4\cdot 0) = 2\tan 0 = 2\cdot 0 = 0$ $\quad$ Point: $(0, 0)$

$x = \frac{\pi}{16}$ $\quad y = 2\tan\left(4\cdot\frac{\pi}{16}\right) = 2\tan\frac{\pi}{4} = 2\cdot 1 = 2$ $\quad$ Point: $\left(\frac{\pi}{16}, 2\right)$

Plot the points and asymptotes. Then, run a curve through the points, approaching the asymptotes on both the left and the right. This will show one period of the function.



20) y = –tan (x – π) 20) ______________

$y = -\tan(x - \pi)$

Relative to the general function, $f(x) = \mathrm{A} \cdot \tan(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = -1, B = 1, C = \pi, D = 0$. Then,

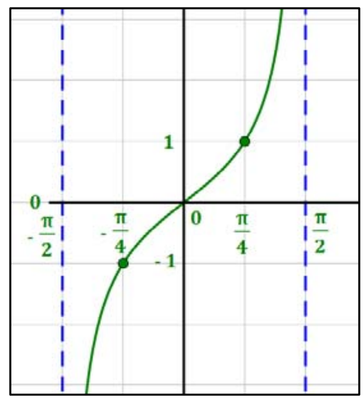
$\text{amplitude} = |\mathrm{A}| = |-1| = 1$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{\pi}{1} = \pi$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = \dfrac{\pi}{1} = \pi$

"$-$" in front of the function indicates a reflection over the x-axis.

Note: This function is shifting the tangent function one full period to the right. Therefore, it is equivalent to the function without a phase shift, i.e., $y = -\tan x$.

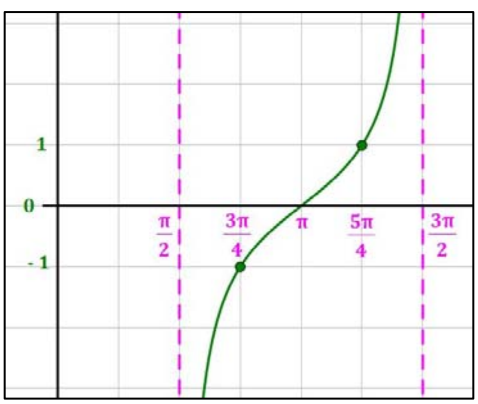


Start: Graph the parent function and its asymptotes.

$y = \tan x$

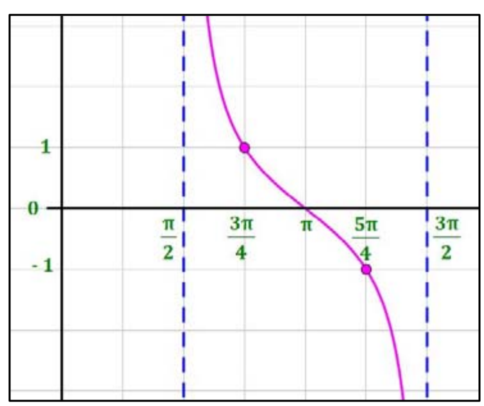
Also, plot the points $\left(-\frac{\pi}{4}, -1\right)$, $\left(\frac{\pi}{4}, 1\right)$

Changes in successive graphs are shown in magenta in the following steps.



Parent period is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. There are no changes to the amplitude or the length of the period because $|A| = 1$ and $B = 1$.

There is a phase shift of π to the right. Adjust the x-axis labels to reflect the shift (add π to each x-axis label and position the labels correctly on the graph).



Reflect the curve over the x-axis because of the minus sign in the front of the equation.

Alternative approach:

20) $y = -\tan(x - \pi)$ 20) ______________

$y = -\tan(x - \pi)$

Relative to the general function, $f(x) = \mathrm{A} \cdot \tan(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = -1, B = 1, C = \pi, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |-1| = 1$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{\pi}{1} = \pi$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = \dfrac{\pi}{1} = \pi$

"$-$" in front of the function indicates a reflection over the x-axis.

> Note: This function is shifting the tangent function one full period to the right. Therefore, it is equivalent to the function without a phase shift, i.e., $y = -\tan x$.

Let's get two asymptotes and three points. Asymptotes of the parent function are: $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$.

Points of interest on the parent function exist at: $x = \left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}\right\}$.

Asymptotes for the plot: Since $B = 1$, start with each asymptote of the parent function and add the phase shift (π in this problem). The resulting asymptotes are:

$$x = \frac{\pi}{2} \qquad \text{and} \qquad x = \frac{3\pi}{2}$$

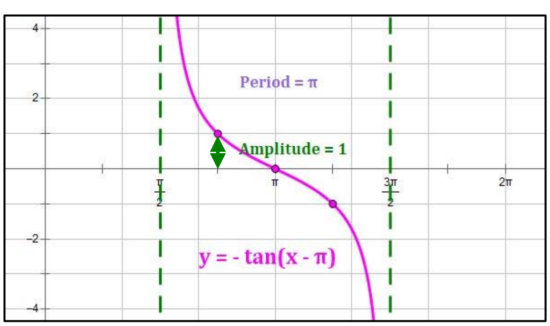
Note that the distance between the asymptotes should be equal to the period.

Points for the plot: Since $B = 1$, start with each x of interest in the parent function and add the phase shift (π in this problem). The resulting x-values of points of interest are: $x = \left\{\frac{3\pi}{4}, \pi, \frac{5\pi}{4}\right\}$.

Now, let's find y-values for each x-value:

$x = \frac{3\pi}{4}$	$y = -\tan\left(\frac{3\pi}{4} - \pi\right) = -\tan\left(-\frac{\pi}{4}\right) = -(-1) = 1$	Point: $\left(\frac{3\pi}{4}, 1\right)$
$x = \pi$	$y = -\tan(\pi - \pi) = -\tan(0) = -(0) = 0$	Point: $(\pi, 0)$
$x = \frac{5\pi}{4}$	$y = -\tan\left(\frac{5\pi}{4} - \pi\right) = -\tan\left(\frac{\pi}{4}\right) = -(1) = -1$	Point: $\left(\frac{5\pi}{4}, -1\right)$

Plot the points and asymptotes. Then, run a curve through the points, approaching the asymptotes on both the left and the right. This will show one period of the function.



> Note: Since this function is equivalent to $y = -\tan x$, we could have drawn the curve for the period between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. It would have the same shape, but would be π units to the left of the curve shown here.

21) y = 4 cot 2x 21) _______________

$y = 4\cot 2x$

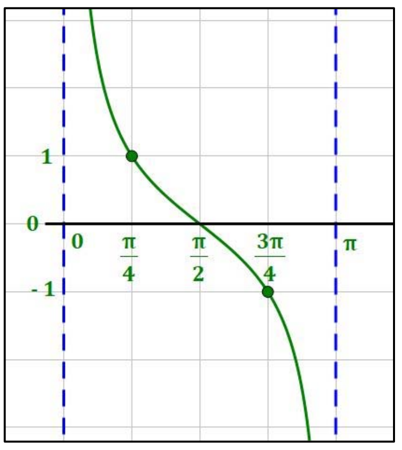
Relative to the general function, $f(x) = \mathrm{A}\cdot\cot(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 4, B = 2, C = 0, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |4| = 4$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{\pi}{2}$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = 0$

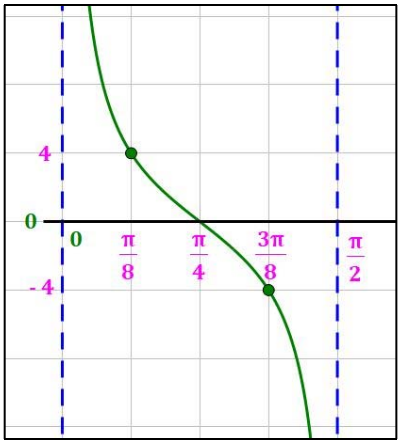


Start: Graph the parent function and its asymptotes.

$y = \cot x$

Also, plot the points $\left(\frac{\pi}{4}, 1\right)$, $\left(\frac{3\pi}{4}, -1\right)$

Changes in successive graphs are shown in magenta in the following steps.



Adjust the amplitude:

- Change the amplitude from 1 to $|A| = 4$ by multiplying the y-axis labels by 4.

Adjust the period:

- Parent period is $[0, \pi]$
- Divide x-axis labels by $B = 2$

Alternative approach:

21) y = 4 cot 2x 21) ____________

$y = 4\cot 2x$

Relative to the general function, $f(x) = \mathrm{A} \cdot \cot(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 4, B = 2, C = 0, D = 0$. Then,

$$\text{amplitude} = |\mathrm{A}| = |4| = 4$$

$$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{\pi}{2}$$

$$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = 0$$

Let's get two asymptotes and three points. Asymptotes of the parent function are: $x = 0, x = \pi$. Points of interest on the parent function exist at: $x = \left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$.

Asymptotes for the plot: Since $B = 2$, divide each asymptote value of the parent function by 2 and add the phase shift (0 in this problem). The resulting asymptotes are:

$$x = 0 \qquad \text{and} \qquad x = \frac{\pi}{2}$$

Note that the distance between the asymptotes should be equal to the period.

Points for the plot: Since $B = 2$, divide each x of interest in the parent function by 2 and add the phase shift (0 in this problem). The resulting x-values of points of interest are: $x = \left\{\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}\right\}$.

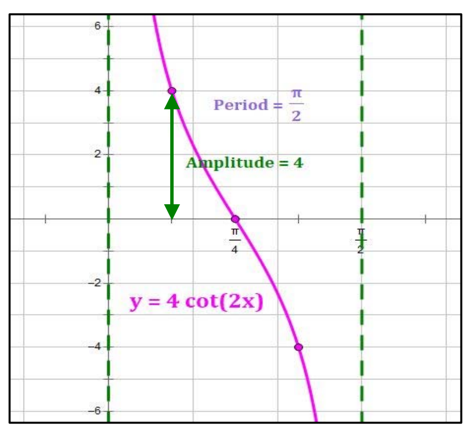
Now, let's find y-values for each x-value:

$x = \frac{\pi}{8}$ $\quad y = 4\cot\left(2 \cdot \frac{\pi}{8}\right) = 4\cot\left(\frac{\pi}{4}\right) = 4 \cdot (1) = 4$ $\quad$ Point: $\left(\frac{\pi}{8}, 4\right)$

$x = \frac{\pi}{4}$ $\quad y = 4\cot\left(2 \cdot \frac{\pi}{4}\right) = 4\cot\left(\frac{\pi}{2}\right) = 4 \cdot (0) = 0$ $\quad$ Point: $\left(\frac{\pi}{4}, 0\right)$

$x = \frac{3\pi}{8}$ $\quad y = 4\cot\left(2 \cdot \frac{3\pi}{8}\right) = 4\cot\left(\frac{3\pi}{4}\right) = 4 \cdot (-1) = -4$ $\quad$ Point: $\left(\frac{3\pi}{8}, -4\right)$

Plot the points and asymptotes. Then, run a curve through the points, approaching the asymptotes on both the left and the right. This will show one period of the function.



22) $y = 3\csc\frac{x}{4}$ 22) ______________

$y = 3\csc\left(\frac{x}{4}\right)$

Relative to the general function, $f(x) = \mathrm{A}\cdot\csc(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 3, B = \frac{1}{4}, C = 0, D = 0.$ Then,

$\text{amplitude} = |\mathrm{A}| = |3| = 3$

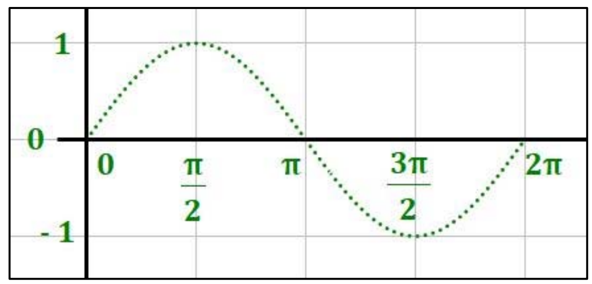
$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{2\pi}{\frac{1}{4}} = 8\pi$

$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = 0$

Start: Graph the reciprocal parent function

$y = \sin x$ (draw it dotted or dashed)

Changes in successive graphs are shown in magenta in the following steps.

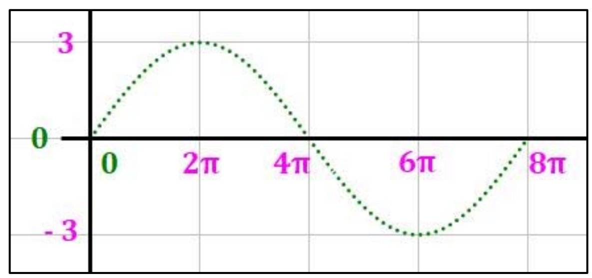


Adjust the amplitude:

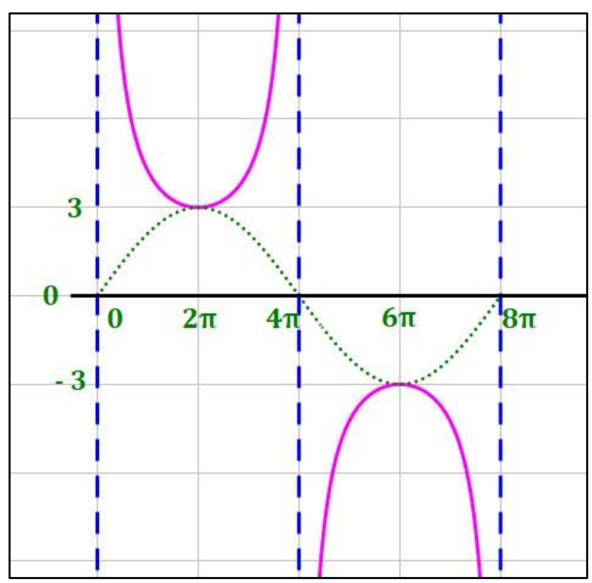
- Change amplitude from 1 to $|A| = 3$
- Change y-axis labels

Adjust the period:

- Parent period is $[0, 2\pi]$
- Divide x-axis labels by $B = \frac{1}{4}$. That is, multiply the x-axis labels by 4.



Add asymptotes for the cosecant function where the sine function has values of zero, and draw U's attached to the sine function between the asymptotes.



Alternative approach:

22) $y = 3\csc\frac{x}{4}$ 22) ____________

$$y = 3\csc\left(\frac{x}{4}\right)$$

Relative to the general function, $f(x) = \mathrm{A}\cdot\csc(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 3, B = \frac{1}{4}, C = 0, D = 0$. Then, $\quad$ $\text{amplitude} = |\mathrm{A}| = |3| = 3$

$$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{2\pi}{\frac{1}{4}} = 8\pi \qquad \text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = 0$$

To plot a cosecant function, first plot the corresponding sine function and the asymptotes of the cosecant function. Then draw the "U's" that make up the cosecant function.

The corresponding sine function for this problem is: $y = 3\sin\left(\frac{x}{4}\right)$. Let's get five points. Points of interest on the parent function exist at: $x = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$. Asymptotes of the parent cosecant function over this interval are: $x = 0,\ x = \pi,\ x = 2\pi$, i.e., where $\sin x = 0$.

Points for the plot: Since $B = \frac{1}{4}$, divide each x of interest in the parent function by $\frac{1}{4}$ (i.e., multiply by 4) and add the phase shift (0 in this problem). The resulting x-values of interest are: $x = \{0, 2\pi, 4\pi, 6\pi, 8\pi\}$. Note that the last x-value should be equal to the period plus the phase shift.

Now, let's find y-values for each x-value:

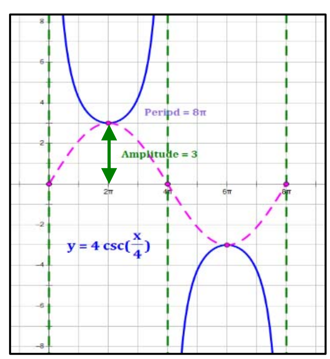
$x = 0$	$y = 3\sin(0 \div 4) = 3\sin 0 = 3\cdot 0 = 0$	Point: $(0, 0)$
$x = 2\pi$	$y = 3\sin(2\pi \div 4) = 3\sin\frac{\pi}{2} = 3\cdot 1 = 3$	Point: $(2\pi, 3)$
$x = 4\pi$	$y = 3\sin(4\pi \div 4) = 3\sin\pi = 3\cdot 0 = 0$	Point: $(4\pi, 0)$
$x = 6\pi$	$y = 3\sin(6\pi \div 4) = 3\sin\frac{3\pi}{2} = 3\cdot(-1) = -3$	Point: $(6\pi, -3)$
$x = 8\pi$	$y = 3\sin(8\pi \div 4) = 3\sin 2\pi = 3\cdot 0 = 0$	Point: $(8\pi, 0)$

Asymptotes for the plot: Since $B = \frac{1}{4}$, divide each asymptote of the parent cosecant function by $\frac{1}{4}$ (i.e., multiply by 4) and add the phase shift (0 in this problem). The resulting asymptotes are:

$x = 0 \quad$ and $\quad x = 4\pi \quad$ and $\quad x = 8\pi$

Note that the distance between the two outside asymptotes should be equal to the period.

Plot the points and asymptotes. Then, plot the "U's" that connect to the sine function at its highest and lowest points, approaching the asymptotes on both the left and the right. This will show one period of the function.



23) y = 3 sec x 23) ______________

$y = 3\sec x$

Relative to the general function, $f(x) = \mathrm{A} \cdot \sec(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 3, B = 1, C = 0, D = 0$. Then,

$\text{amplitude} = |\mathrm{A}| = |3| = 3$

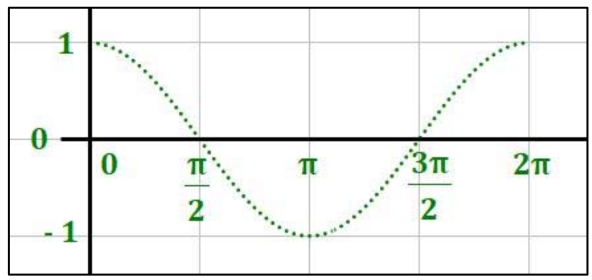
$$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{2\pi}{1} = 2\pi$$

$$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = 0$$

Start: Graph the reciprocal parent function

$y = \cos x$ (draw it dotted or dashed)

Changes in successive graphs are shown in magenta in the following steps.

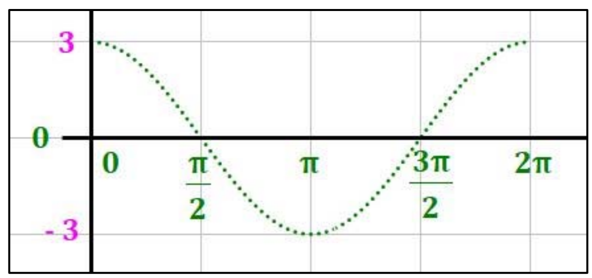


Adjust the amplitude:

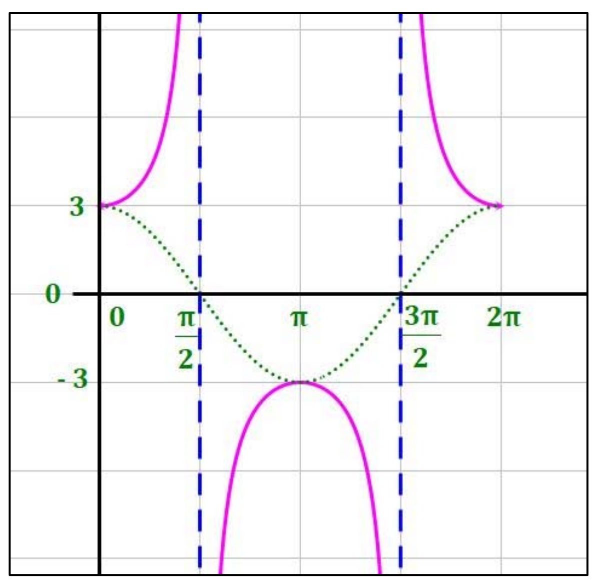
- Change amplitude from 1 to $|A| = 3$
- Change y-axis labels

There is no need to adjust the period or the x-axis labels because $B = 1$

- Parent period is $[0, 2\pi]$



Add asymptotes for the secant function where the cosine function has values of zero, and draw U's or half-U's attached to the cosine function between the asymptotes.



Alternative approach:

23) y = 3 sec x 23) ______________

$y = 3\sec x$

Relative to the general function, $f(x) = \mathrm{A}\cdot\sec(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = 3, B = 1, C = 0, D = 0$. Then, $\text{amplitude} = |\mathrm{A}| = |3| = 3$

$\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{2\pi}{1} = 2\pi$ $\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = 0$

To plot a secant function, first plot the corresponding cosine function and the asymptotes of the secant function. Then draw the "U's" and/or partial "U's" that make up the secant function.

The corresponding cosine function for this problem is: $y = 3\cos x$. Let's get five points. Points of interest on the parent function exist at: $x = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$. Asymptotes of the parent secant function over this interval are: $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, i.e., where $\cos x = 0$.

Points for the plot: Since $B = 1$, add each x of interest in the parent function to the phase shift (0 in this problem). The resulting x-values of interest remain: $x = \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\right\}$. Note that the last x-value should be equal to the period plus the phase shift.

Now, let's find y-values for each x-value:

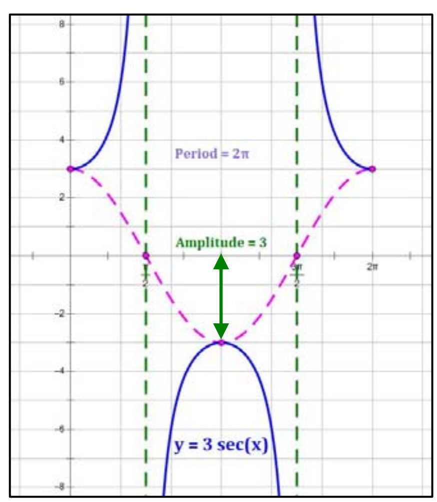
$x = 0$	$y = 3\cos(0) = 3\cdot 1 = 3$	Point: $(0, 3)$
$x = \frac{\pi}{2}$	$y = 3\cos\left(\frac{\pi}{2}\right) = 3\cdot 0 = 0$	Point: $\left(\frac{\pi}{2}, 0\right)$
$x = \pi$	$y = 3\cos(\pi) = 3\cdot(-1) = -3$	Point: $(\pi, -3)$
$x = \frac{3\pi}{2}$	$y = 3\cos\left(\frac{3\pi}{2}\right) = 3\cdot 0 = 0$	Point: $\left(\frac{3\pi}{2}, 0\right)$
$x = 2\pi$	$y = 3\cos(2\pi) = 3\cdot 1 = 3$	Point: $(2\pi, 3)$

Asymptotes for the plot: Since $B = 1$, add each asymptote of the parent secant function to the phase shift (0 in this problem). The resulting asymptotes are unchanged from the parent function:

$$x = \frac{\pi}{2} \quad \text{and} \quad x = \frac{3\pi}{2}$$

Note that the distance between the two asymptotes shown should be equal to half of the period because they only encase one full "U".

Plot the points and asymptotes. Then, plot the "U's" and/or "half-U's" that connect to the cosine function at its highest and lowest points, approaching the asymptotes on both the left and the right. This will show one period of the function.



Inverse Trigonometric Functions

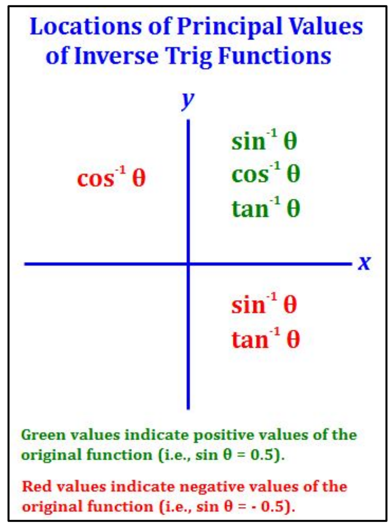
Inverse Trigonometric Functions

Inverse trigonometric functions are shown with a $"-1"$ exponent or an "arc" prefix. So, the inverse sine of x may be shown as $\sin^{-1}(x)$ or $\arcsin(x)$. Inverse trigonometric functions ask the question: which angle θ has a function value of x? For example:

$\theta = \sin^{-1}(0.5)$ asks which angle has a sine value of 0.5. It is equivalent to: $\sin\theta = 0.5$.

$\theta = \arctan(1)$ asks which angle has a tangent value of 1. It is equivalent to: $\tan\theta = 1$.

Principal Values of Inverse Trigonometric Functions



There are an infinite number of angles that answer the above questions, so the inverse trigonometric functions are referred to as multi-valued functions. Because of this, mathematicians have defined a principal solution for problems involving inverse trigonometric functions. The angle which is the principal solution (or principal value) is defined to be the solution that lies in the quadrants identified in the figure at right. For example:

The solutions to the equation $\theta = \sin^{-1} 0.5$ are all x-values in the intervals $\left\{\left(\frac{\pi}{6} + 2n\pi\right) \cup \left(\frac{5\pi}{6} + 2n\pi\right)\right\}$. That is, the set of all solutions to this equation contains the two solutions in the interval $[0, 2\pi)$, as well as all angles that are integer multiples of 2π less than or greater than those two angles. Given the confusion this can create, mathematicians have defined a principal value for the solution to these kinds of equations.

The principal value of θ for which $\theta = \sin^{-1} 0.5$ lies in Q1 because 0.5 is positive, and is $\theta = \frac{\pi}{6}$.

Ranges of Inverse Trigonometric Functions

The ranges of inverse trigonometric functions are generally defined to be the ranges of the principal values of those functions. A table summarizing these is provided at right.

Note: Angles in Q4 are expressed as negative angles.

Ranges of Inverse Trigonometric Functions		
Function	**Range**	**Quadrants**
$\sin^{-1}\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$1, 4$
$\cos^{-1}\theta$	$0 \le \theta \le \pi$	$1, 2$
$\tan^{-1}\theta$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$	$1, 4$

Find the exact value of the expression.

24) $\sin^{-1}\frac{\sqrt{3}}{2}$ 24) ______________

$\sin^{-1}\frac{\sqrt{3}}{2}$ is asking for the angle that has a sine value of $\frac{\sqrt{3}}{2}$. We either have this memorized or we can look it up in the table on page 1 of this document.

$$\sin^{-1}\frac{\sqrt{3}}{2} = \mathbf{60°} = \frac{\boldsymbol{\pi}}{\mathbf{3}}$$

25) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ 25) ______________

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ is asking for the angle that has a cosine value of $\left(-\frac{\sqrt{2}}{2}\right)$. We know from the chart on the prior page that $\cos^{-1}$ has negative values in Q2. So we will determine $\cos^{-1}\left(+\frac{\sqrt{2}}{2}\right)$ in Q1 and then identify the corresponding angle in Q2.

$\cos^{-1}\left(+\frac{\sqrt{2}}{2}\right) = 45°$ In Q2, this corresponds to $(180° - 45°) = 135°$

Then,

$$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \mathbf{135°} = \frac{\mathbf{3}\boldsymbol{\pi}}{\mathbf{4}}$$

26) $\tan^{-1}(1)$ 26) ______________

$\tan^{-1} 1$ is asking for the angle that has a tangent value of 1. We either have this memorized or we can look it up in the table on page 1 of this document.

$$\tan^{-1} 1 = \mathbf{45°} = \frac{\boldsymbol{\pi}}{\mathbf{4}}$$

Find the exact value of the expression, if possible. Do not use a calculator.

27) $\cos^{-1}(\cos \pi)$ 27) ______________

Inverse cosine is defined in Q1 and Q2, up to and including π radians. Therefore, the inverse cosine and cosine functions cancel <u>in these quadrants</u> and we get:

$$\cos^{-1}(\cos \pi) = \boldsymbol{\pi}$$

28) $\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$ 28) ______________

Sometimes these get tricky and you have to do them in 2 steps if the angle in question is not in a quadrant where the inverse function is defined. That is the case here because $\left(-\frac{\pi}{3}\right)$ is in Q4, where the inverse cosine function is not defined. So, in 2 steps:

$$\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right) = \cos^{-1}\left(\frac{1}{2}\right) = \boldsymbol{\frac{\pi}{3}}$$

Use a sketch to find the exact value of the expression.

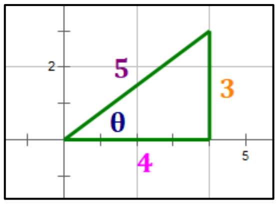
29) $\cos\left(\sin^{-1}\frac{3}{5}\right)$ 29) ______________

$\sin^{-1}\left(\frac{3}{5}\right)$ is an angle in Q1 because the argument, $\frac{3}{5}$, is positive. We are told to sketch a triangle in Q1. Note: we can do this problem without a sketch since we are in Q1 and we know that we are dealing with a 3-4-5 triangle. So, we expect the sketch to show: $\cos\left(\sin^{-1}\left(\frac{3}{5}\right)\right) = \frac{4}{5}$.

Recall that $\sin\theta = \frac{y}{r}$. So, we have $y = 3, r = 5$.

Then, $x = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$.

Finally, from the sketch to the right, $\boldsymbol{\cos\left(\sin^{-1}\left(\frac{3}{5}\right)\right) = \cos(\theta) = \frac{4}{5}}$

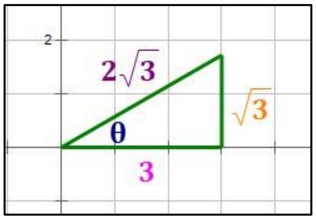


30) $\csc\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$ 30) ______________

$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ is an angle in Q1 because the argument, $\frac{\sqrt{3}}{3}$, is positive.

Recall that $\tan\theta = \frac{y}{x}$. So we have $y = \sqrt{3}, x = 3$.

Then, $r = \sqrt{3^2 + \sqrt{3}^2} = \sqrt{12} = 2\sqrt{3}$.



Next, recall that:

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{y}{r}} = \frac{r}{y}$$

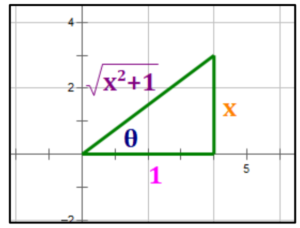
Finally, from the sketch to the right, $\boldsymbol{\csc\left(\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right) = \csc(\theta) = \frac{2\sqrt{3}}{\sqrt{3}} = 2}$

Use a right triangle to write the expression as an algebraic expression. Assume that x is positive and in the domain of the given inverse trigonometric function.

31) $\cos(\tan^{-1} x)$ 31) ______________

We are in Q1 for this problem because the problem states that x is positive. $\tan^{-1} x$ takes positive values of x in Q1 and negative values of x in Q4.

Let's draw a sketch of what we are considering.



$\tan^{-1} x$ is the angle that has a tangent value of x. Since $\tan\theta = \frac{opposite}{adjacent}$, let's draw a triangle with the opposite side x and adjacent side 1. Then,

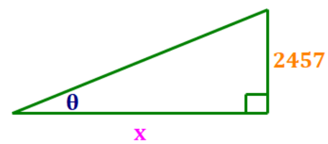
$$r = \sqrt{x^2 + 1^2} = \sqrt{x^2+1}$$

Finally, from the sketch to the right, $\mathbf{\cos(\tan^{-1} x)} = \mathbf{\cos(\theta)} = \frac{\mathbf{1}}{\sqrt{\mathbf{x^2+1}}} = \frac{\sqrt{\mathbf{x^2+1}}}{\mathbf{x^2+1}}$

Solve the problem.

32) From a boat on the river below a dam, the angle of elevation to the top of the dam is 22° 23'. If the dam is 2457 feet above the level of the river, how far is the boat from the base of the dam (to the nearest foot)? 32) ______________

The diagram to the right illustrates this problem. We want to calculate the value of x.

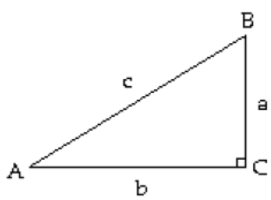


Angle $\theta = 22°\,23' = \left(22 + \frac{23}{60}\right)° \sim 22.383333°$. Then,

$$\tan 22.383333° = \frac{2457}{x} \qquad \Rightarrow \qquad x = \frac{2457}{\tan 22.383333°} = \mathbf{5{,}966\ feet}$$

Note: accuracy of the calculation is an issue in tangent problems, which is why I used six decimals in calculating the tangent value. This will produce an answer accurate to the nearest foot.

Solve the right triangle shown in the figure. Round lengths to one decimal place and express angles to the nearest tenth of degree.



First, let's calculate a.

$$\mathbf{a} = \sqrt{c^2 - b^2} = \sqrt{390^2 - 130^2} = 260\sqrt{2} = \mathbf{367.7}$$

33) b = 130, c = 390 33) ______________

Now let's calculate the angles. You will have the greatest accuracy using the exact values of b and c in your calculations since the value of a is approximate.

$$\mathbf{m\angle A} = \cos^{-1}\frac{b}{c} = \cos^{-1}\frac{130}{390} = \cos^{-1}\frac{1}{3} \sim \mathbf{70.5°}$$

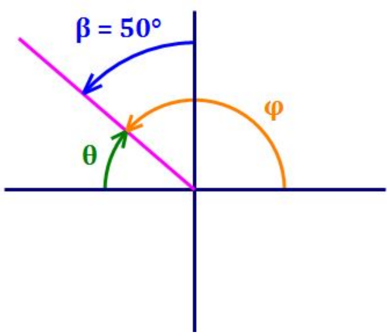
$$\mathbf{m\angle B} = 90° - 70.5° \sim \mathbf{19.5°}$$

$$\mathbf{m\angle C = 90°}$$

Bearings

Bearings are described differently from other angles in Trigonometry. A bearing is a clockwise or counterclockwise angle whose initial side is either due north or due south. The student will need to translate these into reference angles and/or polar angles to solve problems involving bearings.

Some bearings, along with the key associated angles are shown in the illustrations below. The bearing angle is shown as $\boldsymbol{\beta}$, the reference angle is shown as $\boldsymbol{\theta}$, the polar angle is shown as $\boldsymbol{\varphi}$.

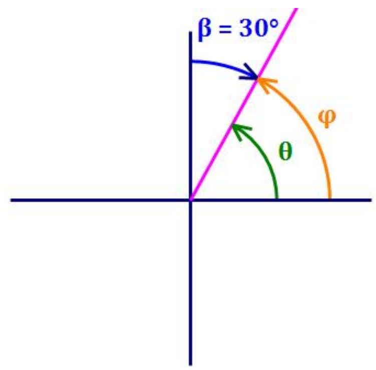


Bearing: $\mathbf{N\ 50°\ W}$

Bearing Angle: $\beta = 50°$

Reference Angle: $\theta = 40°$

Polar Angle: $\varphi = 140°$

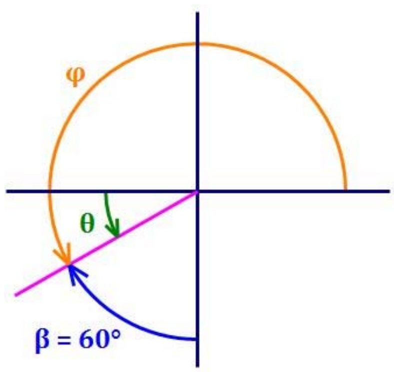


Bearing: $\mathbf{N\ 30°\ E}$

Bearing Angle: $\beta = 30°$

Reference Angle: $\theta = 60°$

Polar Angle: $\varphi = 60°$

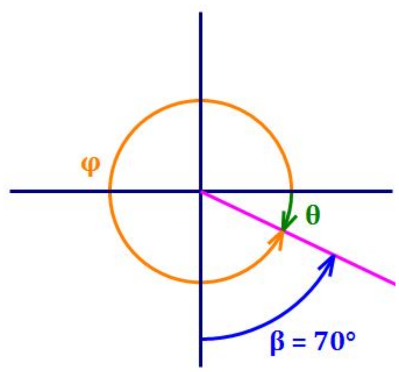


Bearing: $\mathbf{S\ 60°\ W}$

Bearing Angle: $\beta = 60°$

Reference Angle: $\theta = 30°$

Polar Angle: $\varphi = 210°$



Bearing: $\mathbf{S\ 70°\ E}$

Bearing Angle: $\beta = 70°$

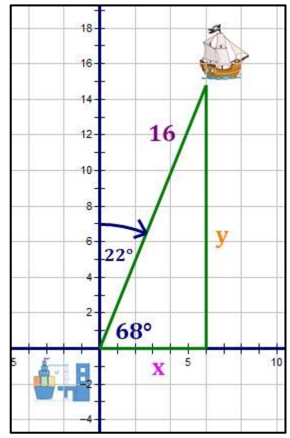
Reference Angle: $\theta = 20°$

Polar Angle: $\varphi = 340°$

Using a calculator, solve the following problems. Round your answers to the nearest tenth.

34) A boat leaves the entrance of a harbor and travels 16 miles on a bearing of N 22° E. How many miles north and how many miles east from the harbor has the boat traveled? 34) ____________

The diagram to the right illustrates this situation. The bearing should be understood as an angle that begins North and moves to the East. However, the angle we need is the complement of that angle. We need:



$\theta = 90° - 22° = 68°$

We will rely on the following formulas for this problem:

$$\left.\begin{array}{lll} \cos\theta = \dfrac{x}{r} & \Rightarrow & x = r\cos\theta \\ \sin\theta = \dfrac{y}{r} & \Rightarrow & y = r\sin\theta \end{array}\right\}$$

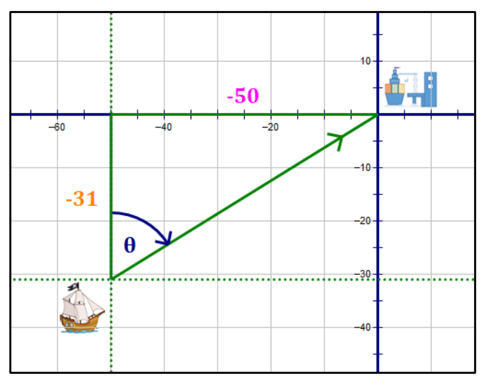
These two formulas are very useful and will be used a lot. So, it's a good idea to memorize them.

Then, the **distance to the east** is: $x = r\cos\theta = 16\cos 68° = \mathbf{6.0}$ **miles**

And, the **distance to the north** is: $x = r\sin\theta = 16\sin 68° = \mathbf{14.8}$ **miles**

35) A ship is 50 miles west and 31 miles south of a harbor. What bearing should the captain set to sail directly to harbor? 35) ____________

The diagram to the right illustrates this situation.



The ship wants to get to the port. We can see by the illustration that we need to calculate the angle θ, and that this angle will give us the bearing in the **NE** direction that the ship needs to travel.

The bearing will be read as an angle that begins North and moves to the East.

$$\tan\theta = \frac{-50}{-31}$$

$$\theta = \tan^{-1}\left(\frac{-50}{-31}\right) = 58.2°$$

Then, **the bearing that the ship must follow is: N 58.2° E**

Simple Harmonic Motion

In Physics, Simple Harmonic Motion is an oscillating motion (think: repeating up and down motion) where the force applied to an object is proportional to and in the opposite direction of its displacement. A common example is the action of a coiled spring, which oscillates up and down when released. Such motion can be modeled by the sine and cosine functions, using the following equations (note: ω is the lower case Greek letter "omega," not the English letter w):

Harmonic motion equations: $d = a\cos\omega t$ or $d = a\sin\omega t$

Period: $\dfrac{2\pi}{\omega}$

Frequency: $f = \dfrac{1}{\text{period}} = \dfrac{\omega}{2\pi}$ or $\omega = 2\pi f$ with $\omega > 0$

Situations in which an object starts at rest at the center of its oscillation, or at rest, use the sine function (because $\sin 0 = 0$); situations in which an object starts in an up or down position prior to its release use the cosine function (because $\cos 0 = 1$).

Example 2.7: An object is attached to a coiled spring**.** The object is pulled up and then released. If the amplitude is 5 cm and the period is 7 seconds, write an equation for the distance of the object from its starting position after t seconds.

The spring will start at a y-value of $+5$ (since it is pulled up), and oscillate between $+5$ and -5 (absent any other force) over time. A good representation of this would be a cosine curve with lead coefficient $a = +5$.

The period of the function is 7 seconds. So, we get:

$$f = \frac{1}{\text{period}} = \frac{1}{7} \quad \text{and} \quad \omega = 2\pi f = 2\pi \cdot \frac{1}{7} = \frac{2\pi}{7}$$

The resulting equation, then, is: $d = 5\cos\left(\frac{2\pi}{7}t\right)$

Example 2.8: An object in simple harmonic motion has a frequency of 1.5 oscillations per second and an amplitude of 13 cm. Write an equation for the distance of the object from its rest position after t seconds.

Assuming that distance $= 0$ at time $t = 0$, it makes sense to use a sine function for this problem. Since the amplitude is 13 cm, a good representation of this would be a sine curve with lead coefficient $a = 13$. Note that a lead coefficient $a = -13$ would work as well.

Recalling that $\omega = 2\pi f$, with $f = 1.5$ we get: $\omega = 2\pi \cdot 1.5 = 3\pi$.

The resulting equations, then, are: $d = 13\sin(3\pi t)$ or $d = -13\sin(3\pi t)$

An object is attached to a coiled spring. The object is pulled down (negative direction from the rest position) and then released. Write an equation for the distance of the object from its rest position after t seconds.

36) amplitude = 8 cm; period = 5 seconds　　　36) ______________

The spring will start at a y-value of -8 (since it is pulled down), and oscillate between -8 and $+8$ (absent any other force) over time. A good representation of this would be a cosine curve with lead coefficient $a = -8$. The form of the cosine oscillation function is: $d = a\cos\omega t$.

The period of the function is 5 seconds. So, we get:

$$f = \frac{1}{\text{period}} = \frac{1}{5} \quad \text{and} \quad \omega = 2\pi f = 2\pi \cdot \frac{1}{5} = \frac{2\pi}{5}$$

The resulting equation, then, is: $d = -8\cos\left(\frac{2\pi}{5}t\right)$

Solve the problem.

37) An object has a frequency of 5 vibrations per second. Write an equation in the form d = sin ωt for the object's simple harmonic motion.　　　37) ______________

Assuming that distance $= 0$ at time $t = 0$, it makes sense to use a sine function for this problem.

We are not given the amplitude for this problem, so let's use "a" for the amplitude. A good representation of this would be a sine curve with lead coefficient a. The form of the sine oscillation function is: $d = a\sin\omega t$.

Recalling that $\omega = 2\pi f$, with $f = 5$ we get: $\omega = 2\pi \cdot 5 = 10\pi$.

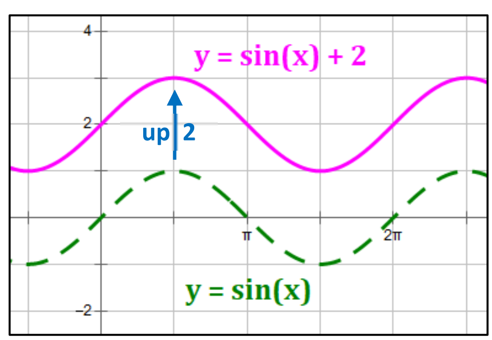
The resulting equation, then, is: $d = \sin(10\pi t)$

In simple harmonic motion problems, the amplitude should always be clearly stated. We are told to write an equation in the form $d = \sin\omega t$. This implies that the amplitude of the vibration is $a = 1$, but that amplitude is not stated in the problem. This is most likely an error on the part of the person who wrote the problem.

In the absence of a stated amplitude, we would typically leave the a as a variable in front of the sine function, e.g., $d = a\sin(10\pi t)$. However, in order to follow the directions of the problem, we must leave out the a. Yuk!

Use a vertical shift to graph the function.

38) y = 2 + sin x　　　38) ______________



Vertical shift is the vertical distance that the midline of a curve lies above or below the midline of its parent function (i.e., the x-axis). For the general function, $f(x) = \mathrm{A} \cdot \mathrm{trig}(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, vertical shift $= \mathrm{D}$.

The value of D may be positive, indicating a shift upward, or negative, indicating a shift downward relative to the graph of the parent function.

For the function, $y = \sin(x) + 2$, we start with the parent function, $y = \sin(x)$, and shift the function up 2. The amplitude and period remain unchanged from the parent function, and there is no phase shift.

39) $y = -4\sin\left(2x + \frac{\pi}{2}\right) - 2$ 39) ______________

$y = -4\sin\left(2x + \frac{\pi}{2}\right) - 2$

Relative to the general function, $f(x) = \mathrm{A}\cdot\sin(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = -4, B = 2, C = -\frac{\pi}{2}, D = -2$. Then,

$\text{amplitude} = |\mathrm{A}| = |-4| = 4$

$\text{period} = \frac{\text{parent function period}}{\mathrm{B}} = \frac{2\pi}{2} = \pi$

$\text{phase shift} = \frac{\mathrm{C}}{\mathrm{B}} = -\frac{\pi}{4} \Rightarrow \frac{\pi}{4}$ to the left

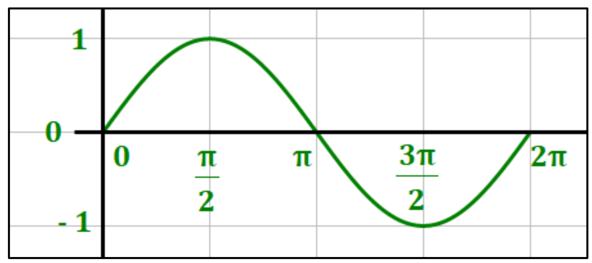
$\text{vertical shift} = \mathrm{D} = -2 \Rightarrow 2\ \text{down}$

"$-$" in front of the function indicates a reflection over the x-axis.

Start: Graph the parent function

$y = \sin x$

Changes in successive graphs are shown in magenta in the following steps.



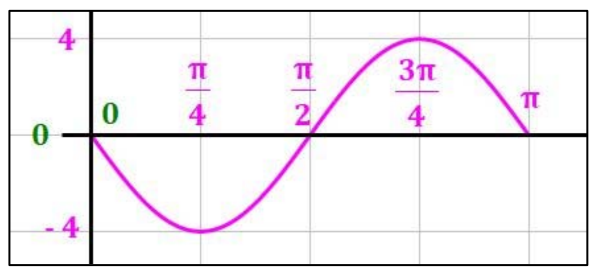
Adjust the amplitude:

- Change amplitude from 1 to $|A| = 4$
- Change y-axis labels

Adjust the period:

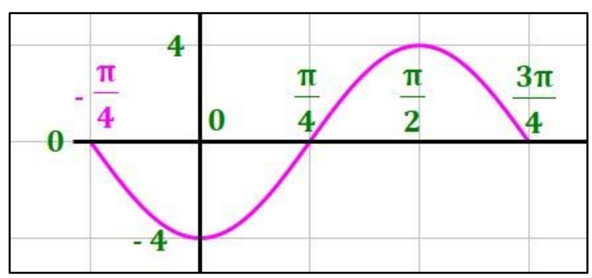
- Parent period is $[0, 2\pi]$
- Divide x-axis labels by $B = 2$

Reflect the curve over the x-axis

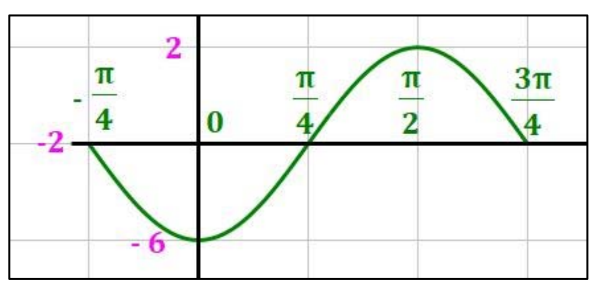


Phase shift the function $\frac{\pi}{4}$ to the left

Also, adjust the x-axis labels to reflect the shift (subtract $\frac{\pi}{4}$ from each x-axis label and position the labels correctly on the graph).



Vertical shift the function down 2 because $D = -2$. To do this, subtract 2 from each y-axis label. Note that we are no longer showing the x-axis. We are showing the midline of the graph in its place.



Alternative approach:

39) $y = -4\sin\left(2x + \frac{\pi}{2}\right) - 2$ 39) ______________

$$y = -4\sin\left(2x + \frac{\pi}{2}\right) - 2$$

Relative to the general function, $f(x) = \mathrm{A}\cdot\sin(\mathrm{B}x - \mathrm{C}) + \mathrm{D}$, we have:

$A = -4, B = 2, C = -\frac{\pi}{2}, D = -2$. Then,

$\text{amplitude} = |\mathrm{A}| = |-4| = 4$ $\quad$ $\text{period} = \dfrac{\text{parent function period}}{\mathrm{B}} = \dfrac{2\pi}{2} = \pi$

$\text{phase shift} = \dfrac{\mathrm{C}}{\mathrm{B}} = -\dfrac{\pi}{4} \Rightarrow \dfrac{\pi}{4}$ to the left $\quad$ $\text{vertical shift} = \mathrm{D} = -2 \Rightarrow 2$ down

"−" in front of the function indicates a reflection over the x-axis.

Let's get five points. Points of interest on the parent function exist at: $x = \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi\}$.

Points for the plot: Since $B = 2$, divide each x of interest in the parent function by 2 and add the phase shift ($-\frac{\pi}{4}$ in this problem). The resulting x-values of interest are: $x = \left\{-\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}\right\}$. Note that the last x-value should be equal to the period plus the phase shift.

Now, let's find y-values for each x-value:

$x = -\frac{\pi}{4}$ $\quad$ $y = -4\sin\left[\left(2\cdot -\frac{\pi}{4}\right) + \frac{\pi}{2}\right] - 2 = -4\sin 0 - 2 = -4\cdot 0 - 2 = -2$ $\quad$ Point: $\left(-\frac{\pi}{4}, -2\right)$

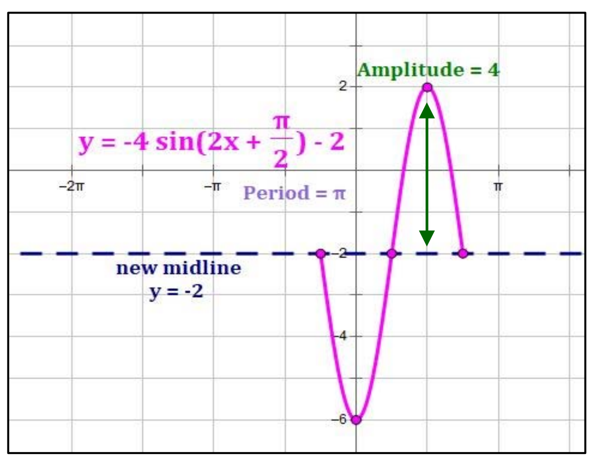
$x = 0$ $\quad$ $y = -4\sin\left[(2\cdot 0) + \frac{\pi}{2}\right] - 2 = -4\sin\frac{\pi}{2} - 2 = -4\cdot 1 - 2 = -6$ $\quad$ Point: $(0, -6)$

$x = \frac{\pi}{4}$ $\quad$ $y = -4\sin\left[\left(2\cdot \frac{\pi}{4}\right) + \frac{\pi}{2}\right] - 2 = -4\sin\pi - 2 = -4\cdot 0 - 2 = -2$ $\quad$ Point: $\left(\frac{\pi}{4}, -2\right)$

$x = \frac{\pi}{2}$ $\quad$ $y = -4\sin\left[\left(2\cdot \frac{\pi}{2}\right) + \frac{\pi}{2}\right] - 2 = -4\sin\frac{3\pi}{2} - 2 = -4\cdot(-1) - 2 = 2$ $\quad$ Point: $\left(\frac{\pi}{2}, 2\right)$

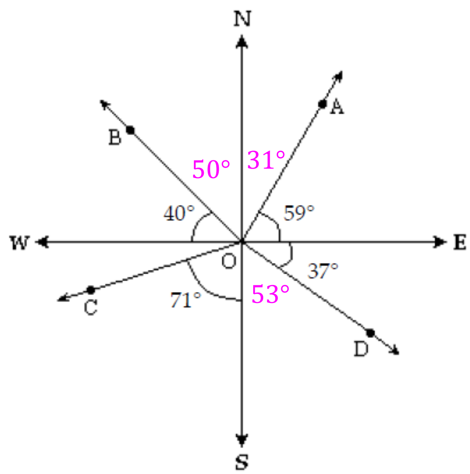
$x = \frac{3\pi}{4}$ $\quad$ $y = -4\sin\left[\left(2\cdot \frac{3\pi}{4}\right) + \frac{\pi}{2}\right] - 2 = -4\sin 2\pi - 2 = -4\cdot 0 - 2 = -2$ $\quad$ Point: $\left(\frac{3\pi}{4}, -2\right)$

Plot the points and run a curve through them. This will show a single wave of the function.



Use the given figure to solve the problem.

40) Find the bearing from O to C., O to D, and O to B. 40) ____________



Bearings start with North or South and move toward East or West. So,

From O to C: **S 71° W**

From O to D: **S 53° E**

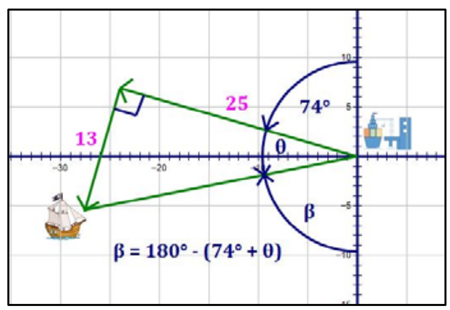
From O to B: **N 50° W**

And, as a bonus bearing:

From O to A: **N 31° E**

Using a calculator, solve the following problems. Round your answers to the nearest tenth.

41) A ship leaves port with a bearing of N 74° W. After traveling 25 miles, the ship then turns 90° and travels on a bearing of S 16° W for 13 miles. At that time, what is the bearing of the ship from port? 41) ____________



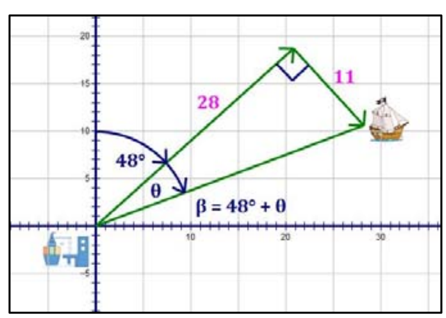
Based on the diagram to the right, we calculate:

$$\theta = \tan^{-1}\left(\frac{13}{25}\right) = 27.5°$$

The total angle, then, from the y-axis to the hypotenuse of the triangle is: $74° + 27.5° = 101.5°$, which puts the location of the ship in Q3, i.e., the SW quadrant.

The bearing of the ship from the port, β, then, is $\text{S } (180° - 101.5°) \text{ W} = \mathbf{S\ 78.5°\ W}$

42) A ship leaves port with a bearing of N 48° E. After traveling 28 miles, the ship then turns 90° and travels on a bearing of S 42° E for 11 miles. At that time, what is the bearing of the ship from port? 42) ____________



Based on the diagram to the right, we calculate:

$$\theta = \tan^{-1}\left(\frac{11}{28}\right) = 21.4°$$

The total angle, then, from the y-axis to the hypotenuse of the triangle is: $48° + 21.4° = 69.4°$, which puts the location of the ship in Q1, i.e., the NE quadrant.

The bearing of the ship from the port, β, then, is: $\mathbf{N\ 69.4°\ E}$